technical proof

TP 3.1
90° rule

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
by David G. Alciatore, PhD, PE ("Dr. Dave")

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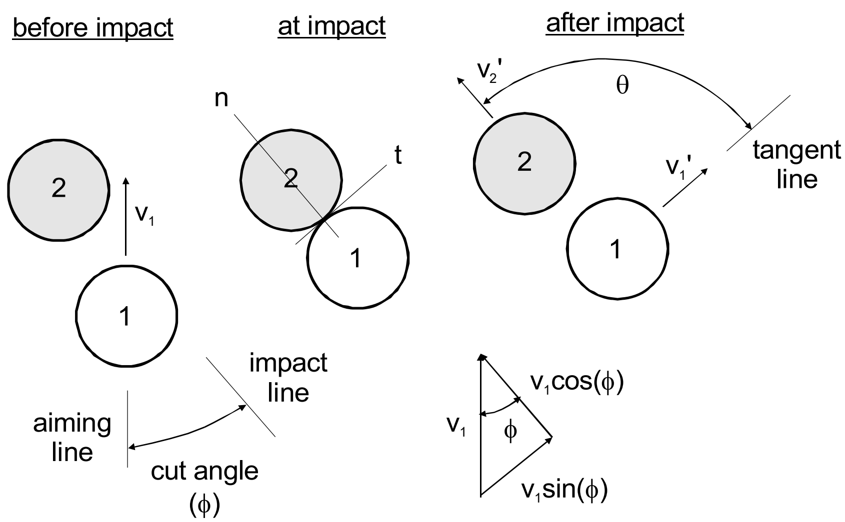

Assumptions:
- the coefficient of friction between the balls is negligible
- the collision is perfectly elastic (the coefficient of restitution is 1)
- both balls have the same mass
- ball 2 is stationary initially

Because there are no forces in the t direction, conservation of linear momentum gives:

$v'_{1t} = v_{1t} = v_1 \cdot \sin(\phi)$ the t component of ball 1's velocity remains constant

$v'_{2t} = v_{2t} = 0$ ball 2 has no t component of velocity before or after impact

Conservation of momentum in the n direction gives:

$$v'_{1n} + v'_{2n} = v_{1n} = v_1 \cdot \cos(\phi) \qquad (1)$$

The coefficient of restitution relation gives:

$$v'_{2n} - v'_{1n} = v_{1n} \qquad (2)$$

the separation speed is equal to the approach speed

Solving Equation 1 and 2 gives:

$v'_{1n} = 0$ ball 1 loses all of its speed in the n direction

$v'_{2n} = v_{1n}$ ball 1 transfers all speed in the n direction to ball 2

After impact, ball 1 has a t component only and ball 2 has an n component only. Therefore,

v'_1 and v'_2 are perpendicular

$$\theta = 90 \cdot \text{deg}$$

<u>Here is a more elegant proof:</u>

Conservation of momentum gives:

$V_1 = V'_1 + V'_2$ (3) where a capital letter denotes a vector

Conservation of energy gives:

$v_1^{\,2} = v'_1{}^2 + v'_2{}^2$ (4) where a lower case letter denotes magnitude

Taking the vector dot product of Equation 3 with itself gives:

$$v_1^{\,2} = v'_1{}^2 + 2 \cdot v'_1 \cdot v'_2 \cdot \cos(\theta) + v'_2{}^2 \qquad (5)$$

Subtracting Equation 4 from Equation 5 gives:

$$2 \cdot v'_1 \cdot v'_2 \cdot \cos(\theta) = 0$$

This can be true only if:

$$\theta = 90 \cdot \text{deg}$$

or if

$v'_1 = 0$ for a head-on stop shot

<u>Even simpler:</u>

The Pythagorian Theorem applied to the triangle representing the vector sum in Equation 3, along with Equation 4, gives the 90 degree result.