CARL ZEISS VISION

Memorandum

TO: Lens Technical Committee Members

CC:

RE: Understanding Reference Wavelengths

Carl Zeiss Vision GmbH

12121 Scripps Summit Drive, Suite 400
San Diego, CA 92131

From: Darryl Meister, ABOM

Title: Manager

Dept: Technical Marketing

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Understanding Reference Wavelengths

The *refractive index* of a transparent material is the ratio of the velocity of light in air (or a vacuum) to the velocity of light in the material. The velocity of light passing through the material actually varies as a function of its electromagnetic wavelength or color. The visible spectrum ranges from roughly 380 to 760 nanometers (nm). Because the atoms of transparent materials have a resonance frequency in the ultraviolet spectrum below 380 nm, light in the blue end of the visible spectrum (toward 380 nm) is actually caused to travel more slowly through the material than light in the red end of the spectrum (toward 760 nm). Essentially, shorter wavelengths of light give up more energy when interacting with the atoms of the material than longer wavelengths, causing light in the blue end to slow more. Consequently, the refractive index of a lens material also varies as a function of wavelength. Blue light has a higher refractive index than red light (Figure 1). White light is therefore "dispersed" into its component colors after refraction through a lens or prism.

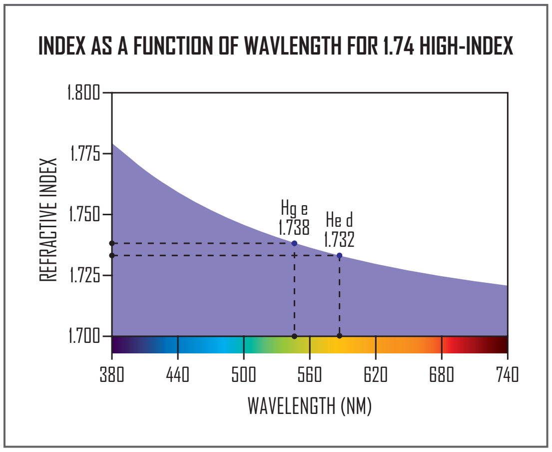

Figure 1. Refractive Index as a Function of Wavelength for 1.74 High-Index

The dispersive property of a lens material is indicated by its *Abbe value* or *constringence*. The Abbe value is essentially a measure of the median refracting action of a material for wavelengths of light across the visible spectrum compared to the difference in refracting action between the red and blue ends of the spectrum. Lens materials with a *lower* Abbe value will generally exhibit a greater difference in refractive index between the red and blue ends of the visible spectrum relative to the median refractive index of the material, resulting in *greater* dispersion. Because the refractive index (n) of a lens material varies as a function of wavelength, the focusing power of an optical lens made from that material also varies as a function of wavelength. The focusing power (F) of a thin lens with a known front surface radius (r_1) and back surface radius (r_2) is given by:

$$F = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Hence, the magnitude of the power of the lens (F) increases as the refractive index (n) of the material increases. The difference in lens focus between the red and blue ends of the visible spectrum due to dispersion is referred to as *chromatic aberration* (Figure 2). Chromatic aberration is proportional to power and prism, so the effects are more pronounced in lenses with higher prescription powers. The dioptric difference in focus (ΔF) between the red and blue ends of the visible spectrum, referred to as *axial* or *longitudinal chromatic aberration*, can be easily determined for a given lens power (F) using the Abbe value (ν):

$$\Delta F = \frac{F}{\nu}$$

Because the refractive index of a lens material varies as a function of wavelength, the focusing power of a lens could vary considerably depending upon the wavelength or color in question. A single color of light must therefore be chosen in order to specify the focusing power of a lens unambiguously. This wavelength or color is known as the *reference wavelength*. Ideally, the reference wavelength should represent a reasonable midpoint within the visible spectrum (near 560 nm) in order to arrive at a single refractive index that is sufficiently close to the average refractive index of the material across the spectrum. The reference wavelength should also correlate well with the peak color sensitivity of the human eye, which is maximally sensitive to wavelengths of light at around 555 nm under *photopic*—or daytime—viewing conditions. The stated refractive index of a lens material is based upon the velocity of the chosen reference wavelength within the material (Table 1).

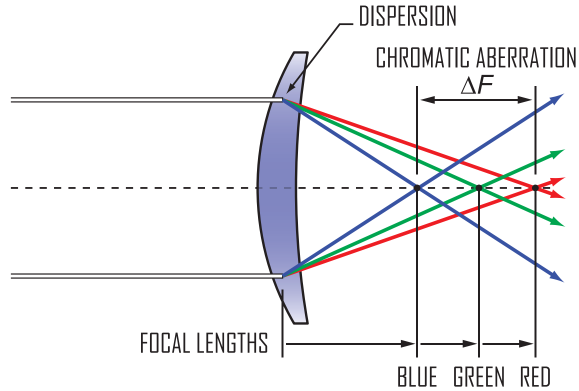

Figure 2. Because of Dispersion, Focal Power Depends Upon the Wavelength (Color) Under Measurement

*Table 1. Optical Properties of Lens Materials**

Lens Material	Index	Abbe
Crown Glass	1.523	59
1.6 Glass	1.601	40
1.7 Glass	1.701	30
1.8 Glass	1.805	25
Hard Resin	1.499	58
Polycarbonate	1.586	30
Trivex®	1.530	44
MR-6	1.595	36
MR-7	1.658	32
MR-8	1.592	41
MR-10	1.661	32
MR-174	1.732	33

** Helium d-line System. Actual values may vary.*
Trivex is a registered trademark of PPG Industries

Since producing purely *monochromatic* (single color) light is difficult in practice, the specific choice of reference wavelength has historically been based upon the ease of isolating the wavelength from polychromatic light. One convenient method of isolating monochromatic light is to excite the gaseous form of an element with electrical current in a gas discharge tube. Elements in a gaseous state will absorb or emit photons of energy that correspond to a set of discrete wavelengths of electromagnetic radiation as the electrons orbiting the nucleus of each atom move from a lower energy state to a higher energy state or vice versa. This allows scientists and engineers to isolate very specific colors of light for measurement. The wavelengths in the visible spectrum associated with these changes in energy state are often referred to as *Fraunhofer lines*, after Joseph Fraunhofer, who carefully measured the narrow bands of color missing from sunlight that had passed through certain gasses (Figure 3).

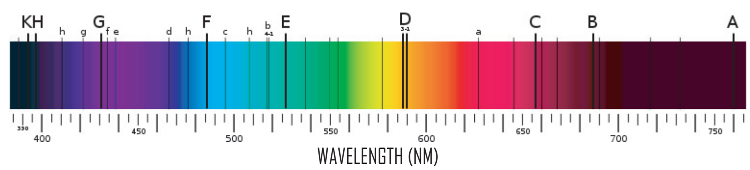

Figure 3. Visible Spectrum with Common Spectral Lines Indicated

Originally, most countries used a reference wavelength to determine the refractive index of lens materials that corresponds to the "Sodium D" (Na D) Fraunhofer line at 589.29 nm, a yellowish spectral color emitted by excited sodium gas. The Sodium D-line actually represents the average of two very closely spaced Fraunhofer lines (D1 and D2) at 589.56 and 589.00 nm, respectively. By the 1980s, two new reference wavelengths were in widespread use in many countries. A yellowish wavelength that corresponds to the "Helium d" (He d) Fraunhofer line at 587.56 nm was adopted by many countries, including the United States, the United Kingdom, and Australia. Several other countries, on the other hand, including Germany, France, and Japan, adopted a greenish wavelength that corresponds to the "Mercury e" (Hg e) Fraunhofer line at 546.07 nm.

Both of these reference wavelengths were eventually standardized in the 1984 edition of the ISO 7944 standard on "Optics and optical instruments." Optical manufacturers now utilize one reference wavelength or the other, depending upon their region. There are technical and practical considerations associated with the choice of either reference wavelength system. The wavelengths used to define the Abbe value also differ slightly between the two reference wavelength systems, although the differences are generally negligible. Although the use of a single reference wavelength globally would obviously be desirable, the costs and training involved in recalibrating all ophthalmic diagnostic and testing equipment for a new reference wavelength have proven prohibitive for some countries.

It is possible to estimate the refractive index of a lens material at a given wavelength using various "dispersion" equations. Such equations can be used, for instance, to estimate the refractive index of a lens material at 546.07 nm in the Mercury e-line system when only the Abbe value and refractive index at the Helium d-line reference wavelength are known. One common dispersion equation is *Cauchy's equation*, which is a refractive index function of wavelength (λ) having the form:

$$n(\lambda) = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4} \ldots$$

Unfortunately, dispersion equations require knowledge of the refractive index of the lens material at multiple wavelengths in order to determine the coefficients of the series (*a*, *b*, *c*...). Only the Abbe value and the refractive index of the lens material at the Helium d-line are commonly available. The Abbe value is based indirectly, however, on a calculation that involves the refractive index of the material at three specific wavelengths (656.27, 486.13, and 587.56 nm). Consequently, it is possible to derive a more useful form of Cauchy's equation based upon the Abbe value (ν) and refractive index (*n*) of a lens material by solving for the coefficients of the first two terms of Cauchy's expression (*a* and *b*) using simultaneous equations:

$$n(\lambda) = n + \frac{n-1}{\nu}\left(\frac{523{,}655}{\lambda^2} - 1.5168\right)$$

For example, MR-174 is a high-index lens material invented by Mitsui Chemicals with a refractive index of 1.732 using the Helium d-line system and an Abbe value of 33. The refractive index using the Mercury e-line system (λ = 546.07 nm) is given by:

$$n(546.07) = 1.732 + \frac{1.732-1}{33}\left(\frac{523{,}655}{546.07^2} - 1.5168\right) = 1.737$$

The calculated index agrees very closely with the manufacturer's stated refractive index of 1.738 for this lens material. Differences between reference wavelength systems have periodically resulted in some degree of confusion in the marketplace. For instance, the very same high-index resin may be described as either a "1.66" high-index material or a "1.67" high-index material, depending upon the reference wavelength used. This is a common occurrence when lens materials developed in one country are distributed in another country that uses a different reference wavelength system, since the refractive index of a lens material is always slightly higher when expressed using the Mercury e-line system compared to the Helium d-line system.

Refractor-head and trial-frame lenses are calibrated to a specific reference wavelength system. If the refractive index of a lens material at the correct reference wavelength is not utilized when calculating the actual lens surfaces for fabrication, errors from the intended prescription powers will occur that may reduce vision quality. Additionally, when measuring lens power, it is equally important to adhere to the correct reference wavelength system. Because two different reference wavelength systems are in common use, automatic focimeters that measure power will often have configuration settings that set the default reference wavelength of the device. Furthermore, because these devices often estimate the power of the lens at the reference wavelength based upon the measurement of the focus of a light source that emits a different wavelength, such as a red LED with peak output above 600 nm, the dispersive property of the lens material may influence the power reading. Some automatic focimeters also have settings for the Abbe value of the lens material in order to compensate for this effect.

Fortunately, the errors in power measurement due to the difference between the two reference wavelength systems are generally small, typically less than 1% of the actual power. Nevertheless, for high-powered lenses made from materials with relatively low Abbe values, like polycarbonate and high-index, measurement errors can reach up to ±0.12 diopters or more (Figure 4). Moreover, even in lower powers, any *spurious* errors in measurement due to the use of the wrong device settings can exacerbate any *real* errors in manufacturing, even when these errors are small, causing lenses that may be perfectly acceptable to appear to fail power inspection in some cases due to the propagation of these errors. An inherent measurement bias of 1% as a result of the use of the wrong reference wavelength will exhaust up to half of the ANSI Z80.1 tolerance on lens power. Conversely, lenses that appear acceptable may in reality exceed recommended power tolerances in the presence of this systematic measurement bias.

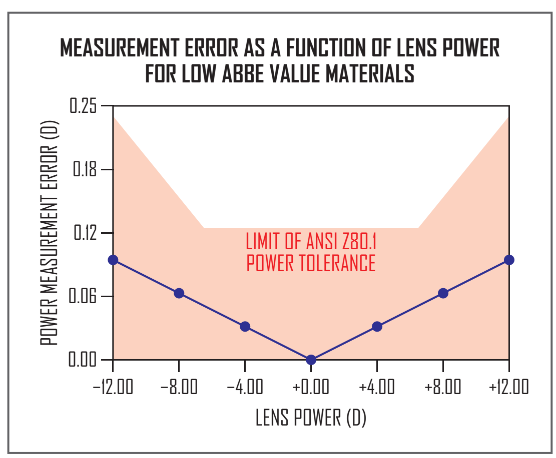

Figure 4. Measurement Error Due to Difference Between Reference Wavelengths as a Function of Lens Power