

CHAPTER 11

FRACTURE OF MATERIALS

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11.1 Brittle vs. Ductile Fracture

Fracture involves the forced separation of a material into two or more parts. Brittle Fracture involves fracture without any appreciable plastic deformation (i.e. energy absorption). Ductile Fracture in the converse and involves large plastic deformation before separation. The difference between brittle and ductile fracture is illustrated in figures 1 and 2. Remembering that the area under the $\sigma - \epsilon$ curve, Fig. 1, represents energy, we can see that much less energy is expended in brittle fracture than in ductile fracture.

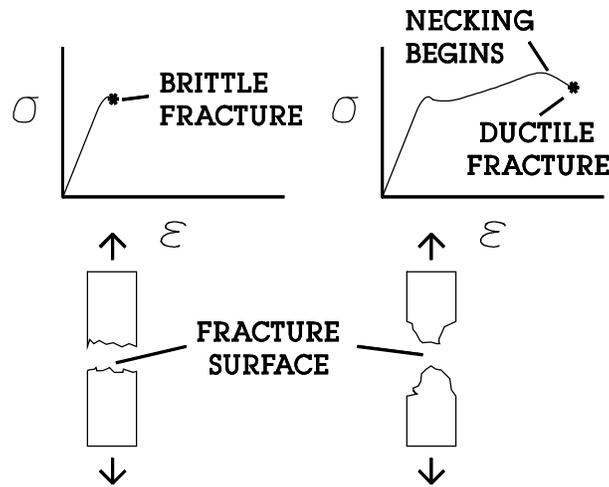


Figure 1: Brittle vs Ductile Fracture

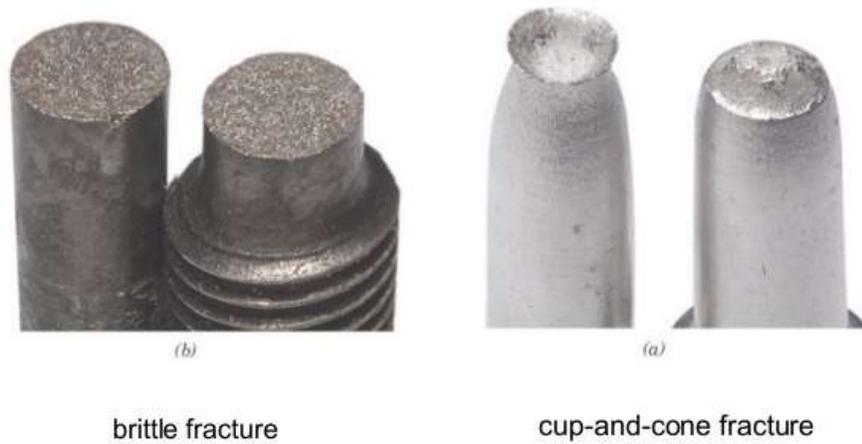


Figure 2. Photos of Tensile Specimens (Left) Brittle, (right) Ductile

Toughness is a measure of a material's ability to absorb energy before failure. Formally, the toughness is the integral under the stress-strain curve up to the fracture point. The Charpy Impact test is often used to determine the material toughness (or relative fracture performance) in the presence of notches and temperature changes. This test is illustrated in figure 3, along with typical result for steel and aluminum.

The figure also shows the temperature effect. Reducing the temperature below the "transition temperature" causes the material to change from ductile to brittle. For example, during World War II, Liberty Ships operating in the North Atlantic had failures due to brittle fracture because the water temperature was below the transition temperature of the material (Fig. 4).

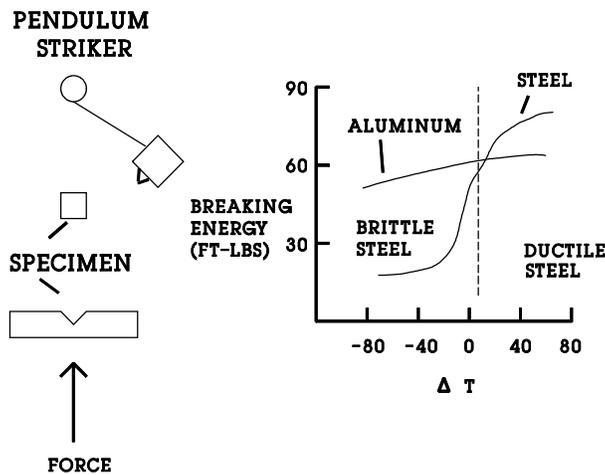


Figure 3: Charpy Impact Test

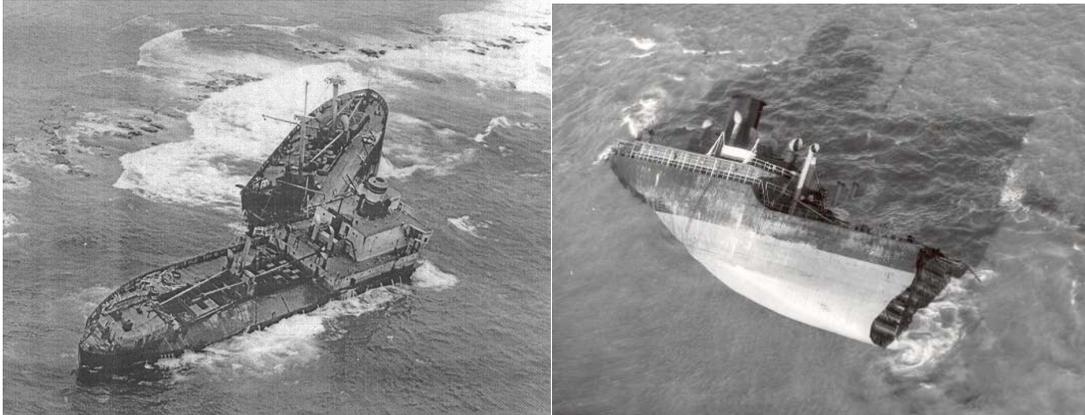


Figure 4. Fractured Hull of Liberty Ship

Brittle fracture is generally governed by the maximum tensile principal stress. A simple illustration is a piece of chalk in torsion (Fig 5). Here the plane of the principal tensile stress is at 45° to the axis of the chalk, and thus is the plane of the fracture.

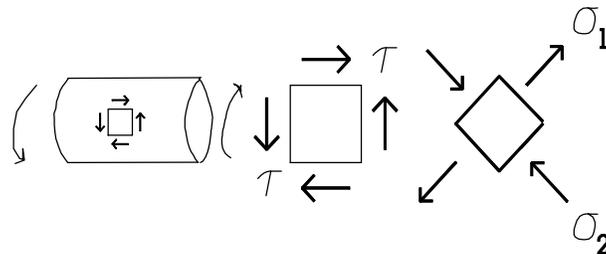


Fig 5: Principal Tensile Stress for Chalk in Torsion

11.2 Temperature Effects and Fracture Analysis Diagram

We have noted earlier in connection with the Charpy impact test that the fracture of steel is very sensitive to the testing temperature within certain temperature ranges. To illustrate this point further, consider a material with a certain crack size present. Below a certain temperature, called the Nil Ductility transition Temperature (NDT temperature), the fracture will be completely brittle in nature. At temperature greater than the NDT temperature, some plastic deformation will accompany the fracture and the fracture will be ductile to some degree. The stress required to cause brittle fracture will be less than that to cause ductile fracture in the same material and the fracture stress vs temperature behavior will be as indicated in figure 25 for a given crack size.

If we consider a number of crack sizes, we will generate a family of curves similar to that shown in figure 6. This leads to the Fracture Analysis Diagram. Such a diagram is shown in Figure 7 for a low carbon construction steel. The vertical axis in the fracture analysis diagram represents the operating stress and the horizontal axis represents the operating temperature, expressed as:

$$T = T_{NDT} + \Delta T \quad \text{therefore, } T - T_{NDT} = \Delta T$$

with the origin denoting $\Delta T = 0$.

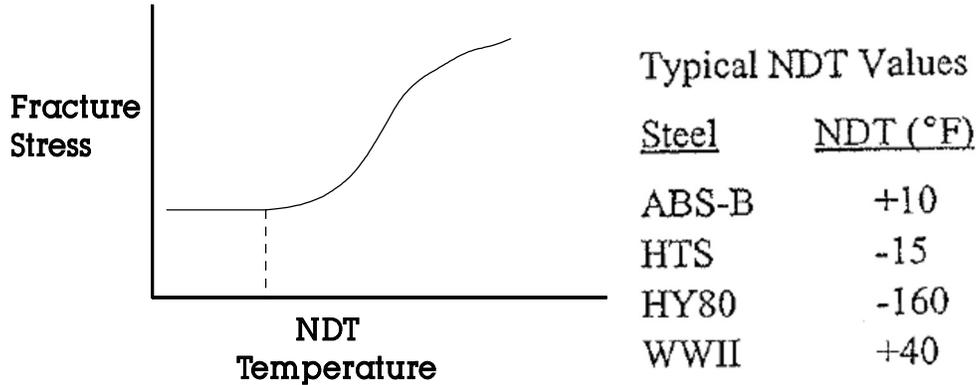


Figure 6 Illustration of Typical Fracture Behavior for Steels

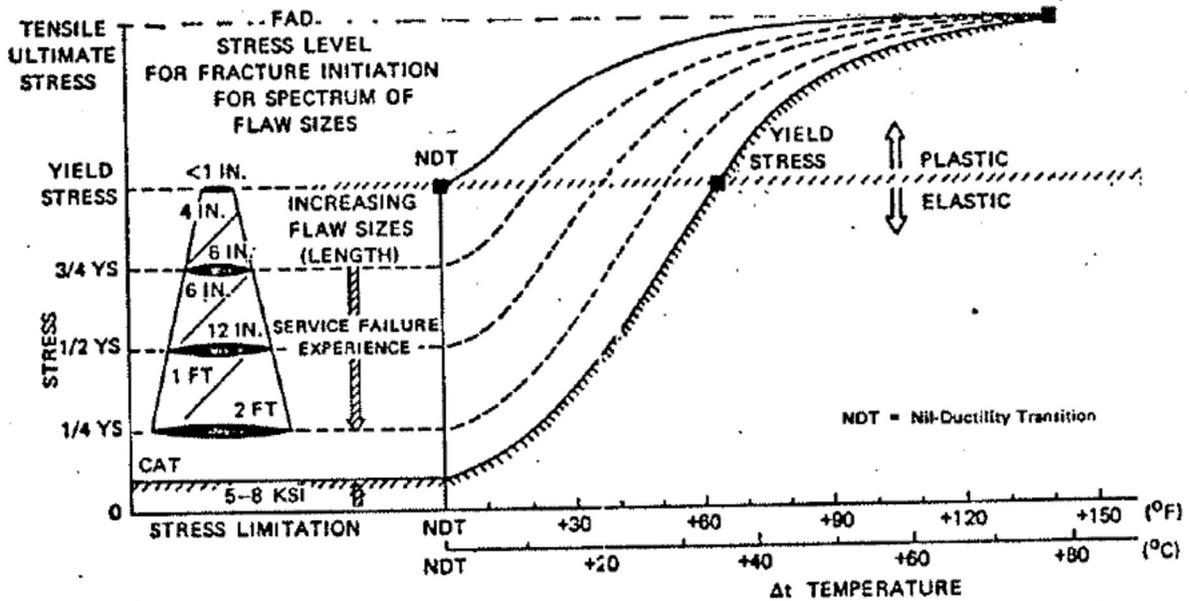


Figure 7: Fracture Analysis Diagram

Example. The walls of a submersible are to be made of a steel having a yield stress of 45 ksi. For this material, under a nominal stress of 30 KSI and an operating temperature of 35°F, determine the NDT temperature where a "stress concentration factor" of 1.5 exists, if (a) 8 inch cracks are to be arrested and (b) if 12 inch cracks are to be arrested.

Solution. Because of the stress concentrations the operating stress will be $(1.5)(30) = 45$ ksi, which equals the yield stress. Reading horizontally across from the yield-stress value in Figure 28 to the intersection of the dotted curve for 8 inch cracks we find the temperature $NDT + 25$. This must equal the operating temperature of $35^{\circ}F$ so the required NDT temperature is $10^{\circ}F$. Similarly for 12 inch cracks, $NDT = 0^{\circ}F$.

Example. For a steel with an operating stress equal to $1/2$ of the yield stress, determine the smallest crack size which could cause brittle fracture at $40^{\circ}F$ if the steel has a NDT of $15^{\circ}F$.

Solution. Reading horizontally in Figure 28 at a stress level of $1/2$ yield stress and vertically up from a temperature of $NDT + 25^{\circ}F$, we find the intersection at an interpolated fracture curve of about 18 inches. Hence cracks of 18 inches or longer would result in fracture.

11.3 Griffith Theory of Brittle Fracture.

Consider a thin plate of length l having a thru-crack of length $2c$, as shown in figure 8.

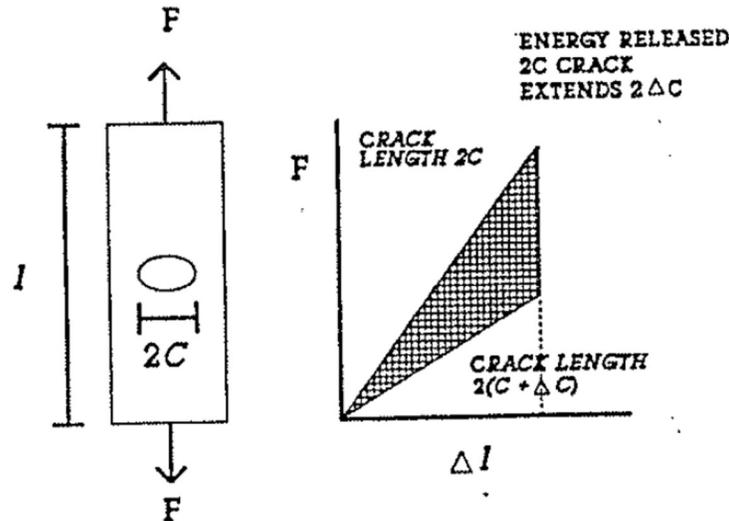


Fig. 8 Fracture with Thru Crack

For a non-extending crack of length $2c$, the force-deflection curve will be as the upper curve shown in Figure 8. For a non-extending crack of length $2(c + \Delta c)$, the curve will be as the lower curve in Figure 8. The area between these curves thus represents the energy released as the crack extends from $2c$ to $2(c + \Delta c)$.

Using elasticity theory Griffith showed that the energy **released** per unit thickness during a crack growth of $2\Delta c$ is

$$\Delta w_e = \frac{2\pi\sigma^2}{E}c\Delta c$$

The creation of additional crack surface **requires** surface energy per unit thickness given by

$$\Delta w_s = 2\gamma_s(2\Delta c) = 4\gamma_s\Delta c$$

where γ_s is the surface energy per unit area.

Now, if $\Delta w_e < \Delta w_s$ the crack will not grow since the released energy will be less than that required to create a new surface.

If, on the other hand, $\Delta w_e \geq \Delta w_s$, crack growth will occur since adequate energy is available for creating the new surface. Hence for crack growth and subsequent failure we must have the condition

$$\frac{2\pi\sigma^2}{E}c\Delta c \geq 4\gamma_s\Delta c$$

The critical fracture stress σ_c is therefore

$$\sigma_c = \sqrt{\frac{2E\gamma_s}{\pi c}}$$

Thus, the critical stress is inversely proportion to $c^{1/2}$. Hence, the smaller the flaw, the greater the value of σ_c . (The entire concept of crack growth is based on the natural tendency for things (midshipmen included) to seek the lowest energy state available. Thermodynamically, there is a 'hump' which must be overcome in order for this to occur. In the case of crack propagation the energy supplied during the loading of the material must be sufficient enough to overcome the 'hump').

The Griffith theory is good for very brittle material, such as glass, in which no plastic deformation accompanies the fracture. When there is some plastic deformation associated with the crack extension, we must add the plastic work γ_p expended in making the surface to the surface energy term γ_s to obtain

$$\sigma_c = \sqrt{\frac{2E(\gamma_s + \gamma_p)}{\pi c}}$$

This formula forms the starting point for modern fracture mechanic analysis as discussed below.

11.4 Fracture mechanics of high-strength materials

High strength steels (yields stresses of 100,000 psi or greater) have a low toughness, and designs using these materials must employ fracture mechanics methods of analysis to insure against failure by brittle fracture. Using the Griffith formulation, we define a new property of the material:

$$\sigma_c \sqrt{\pi c} = k_c$$

where K_c is known as the fracture toughness of the material and is equal to

$$k_c = \sqrt{2E(\gamma_s + \gamma_p)}$$

and has the units of

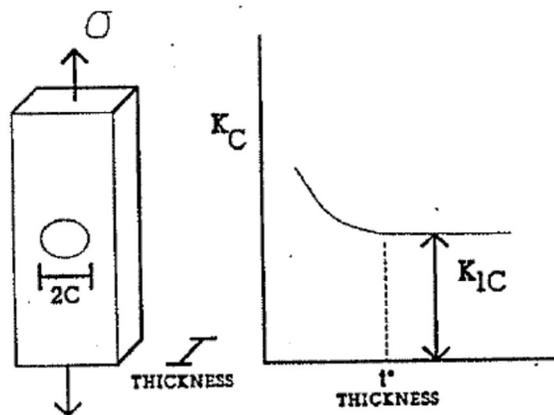
$$\frac{lbf}{inch^2} \sqrt{inch}$$

The quantity K_c can be measured in a simple test by simply measuring the size of an induced crack and measuring the resulting fracture stress, as shown in Figure 9(a). Figure 9(b) shows the effect of varying the thickness of the test plate. It will be seen that K_c falls of as plate thickness increases and reaches a constant value of K_{IC} at thickness t^* . This critical thickness, t^* , varies with the material under consideration but usually is 1/2 inch or less. In using equation for an interior crack, it is always conservative to take $K_c = K_{IC}$. The variation of K_{IC} with the yield strength for steels is illustrated in figure 10.

The critical defect size is

$$C_c = \frac{K_{IC}^2}{\pi \sigma^2}$$

where σ is the applied stress and $2C_c$ is the flaw or crack size needed for brittle fracture. For surface and interior cracks that do not extend through the plate thickness, we have the geometry shown in figure 11.



(a) Figure. 9 (b)

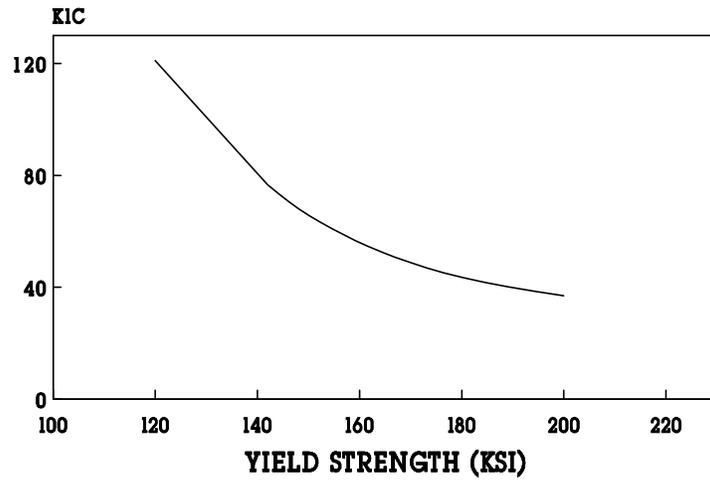


Figure 10: Fracture Toughness vs Yield Strength for Steel

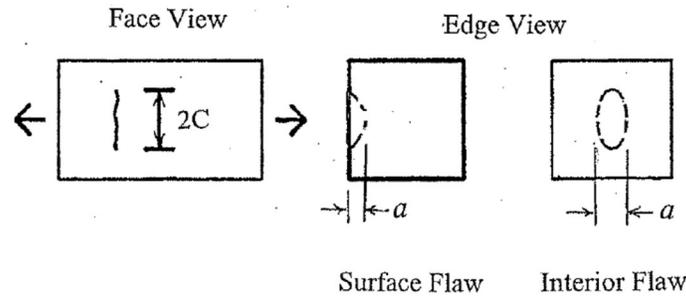


Fig. 11 Flaw Geometry

Typical Fracture-Toughness Values for Selected Engineering Alloys

Material	K_{Ic}		$\sigma_{yield strength}$	
	MPa \sqrt{m}	ksi \sqrt{in}	MPa	ksi
Aluminum alloys:				
2024-T851	26.4	24	455	66
7075-T651	24.2	22	495	72
7178-T651	23.1	21	570	83
Titanium alloy:				
Ti-6Al-4V	55	50	1035	150
Alloy steels:				
4340 (low-alloy steel)	60.4	55	1515	220
17-7 pH (precipitation hardening)	76.9	70	1435	208
350 maraging steel	55	50	1550	225

Approximate formulas for the critical flaw size for cracks which do not initial extend thru the plate thickness are:

for a **surface crack**:

$$a_c = \frac{K_{IC}^2}{\pi\sigma^2} \left[\frac{\phi^2 - 0.2(\sigma/\sigma_y)^2}{1.2} \right]$$

for an **interior crack**:

$$a_c = \frac{K_{IC}^2}{\pi\sigma^2} [\phi^2 - 0.2(\sigma/\sigma_y)^2]$$

where ϕ depends on the ratio a/c as given below.

a/c	ϕ
0	1.0
0.2	1.05
0.4	1.15
0.6	1.28
0.8	1.42
1.0	1.57

Example. For a "thumb-nail" surface flaw, $a/c = 0.4$. Hence $\phi = 1.15$. Suppose the operating stress to be equal to $\frac{1}{2}\sigma_y$, we thus have the critical flaw size for fracture given as

$$a_c = \frac{K_{IC}^2}{\pi\sigma^2} \left[\frac{(1.15)^2 - (0.2)(0.5)^2}{1.2} \right] = 1.06 \frac{K_{IC}^2}{\pi\sigma^2}$$

Example. A pressure vessel is to be made using a steel with $K_{IC} = 41 \text{ ksi} \cdot (\text{in})^{1/2}$ and $\sigma_y = 220 \text{ ksi}$. Assuming surface cracks of 0.05 inches or less and $\sigma = \sigma_y$, determine whether the chosen material is adequate from fracture considerations.

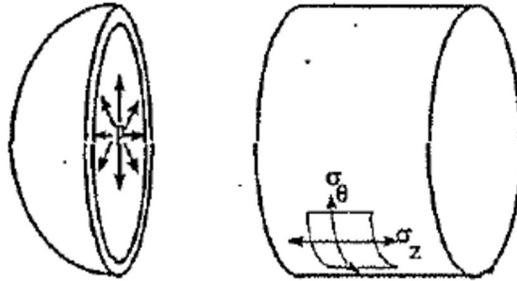
Assume first that $a/c = 0.4$ (thumb nail flaw). then $\phi = 1.15$ and $\sigma = \sigma_y$, we have

$$K_{IC}^2 = \frac{1.2 a_c \pi \sigma_y^2}{\phi^2 - 0.2} = 8130$$

Hence $K_{IC} = 90 \text{ ksi} \cdot \text{in}^{1/2}$. But for the material, K_{IC} is only equal to $41 \text{ ksi} \cdot \text{in}^{1/2}$. Thus fractures will occur for thumb-nail cracks.

Next assume $a/c = 1.0$; we then find that $K_{IC} = 66 \text{ ksi-in}^{1/2}$. This is still higher than K_{IC} for the material. Hence to ensure against fracture, we must select a material with a greater fracture toughness or reduce the operating stress.

Example. A pressure vessel has material properties $\sigma_y = 65 \text{ ksi}$ and $K_{IC} = 35 \text{ ksi-in}^{1/2}$. If the diameter is 20 in and the wall thickness is 1 inch, determine the maximum pressure the tube can withstand before failure if thumbnail cracks of depth 0.1 inches exist.



Solution:

$$\sigma_\theta = \frac{Pa}{t} = 10p, \quad \sigma_z = \frac{Pa}{2t} = 5p$$

First check the maximum pressure required for yielding. The Mises condition gives

$$\sigma_\theta^2 + \sigma_z^2 - \sigma_\theta\sigma_z = \sigma_y^2$$

Hence

$$75p^2 = \sigma_y^2$$

and

$$p = 7.50 \text{ ksi (for yielding).}$$

Next check the maximum pressure for fracture. Here only σ_θ enters since fracture is determined by the greatest tensile principal stress. We have

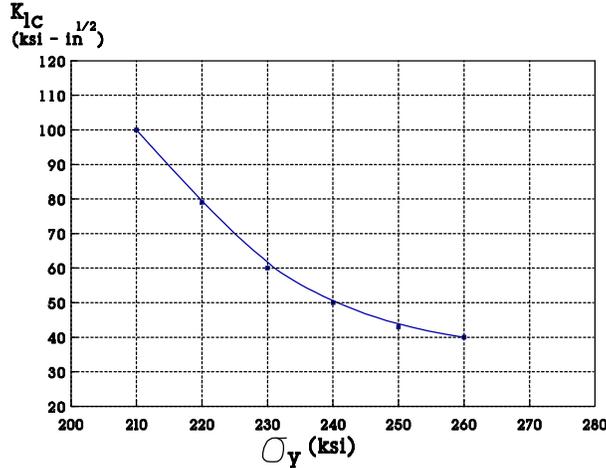
$$K_{ic}^2 \left[\phi^2 - 0.2 \left(\frac{\sigma_\theta}{\sigma_y} \right)^2 \right] = 1.2a\pi\sigma_y^2 \left(\frac{\sigma_\theta}{\sigma_y} \right)^2$$

$$1620 - 245 \left(\frac{\sigma_\theta}{\sigma_y} \right)^2 = 1593 \left(\frac{\sigma_\theta}{\sigma_y} \right)^2$$

$$\frac{\sigma_\theta}{\sigma_y} = 0.939$$

Therefore, fracture will occur when $\sigma_\theta = 0.939 \sigma_y$ or when $p = 0.1(0.939)\sigma_y$ and $p = 6.10$ ksi which is before the yielding.

Example. A thin-walled cylindrical pressure vessel is to have a wall thickness of 0.5 inches and a radius of 10 inches. The material used has K_{IC} vs σ_y fracture characteristics as shown. Surface cracks in the material may be assumed no deeper than 0.025 inches. Select the appropriate σ_y value which will allow the greatest safe pressure loading. What is the pressure value?



Solution:

Fracture Characteristics for Example

We assume thumbnail flaws with $a/c = 0.4$ and $\phi = 1.15$. From the surface flaw equation we have

$$K_{ic}^2 = \frac{1.2a\pi\sigma_\theta^2}{\phi^2 - 0.2\left(\frac{\sigma_\theta}{\sigma_y}\right)^2}$$

From the Mises yield condition we also have

$$\sigma_\theta^2 + \sigma_z^2 - \sigma_\theta\sigma_z = \sigma_y^2$$

Therefore for yielding we have (with $\sigma_\theta = 20p$, $\sigma_z = 10p$)

$$p = 0.0577\sigma_y$$

Constructing the following table:

σ_y	P	σ_θ	K_{ic}	Remarks
200	11.55	231	68.8	too low
240	13.85	277	82.4	too high
220	12.69	254	75.6	ok

Thus at $\sigma_y = 220$ ksi the allowable pressure is 12.69 ksi.

11.5 Connection with Fracture Mechanics

The above results imply, in terms of modern fracture mechanics, that the "fracture toughness" (or critical stress intensity factor) varies with the relative temperature $T - T_{NDT}$. Such variation has been given in the ASME (American Society of Mechanical Engineers) Boiler and Pressure Vessel Code used in the design of such vessels.

This variation is shown in Figure 12 for steels used in pressure vessels such as containment vessels for nuclear reactors. (Yield stresses $\approx 50 - 70$ ksi at room temperature). The values plotted are the minimum values from individual tests showing scatter.

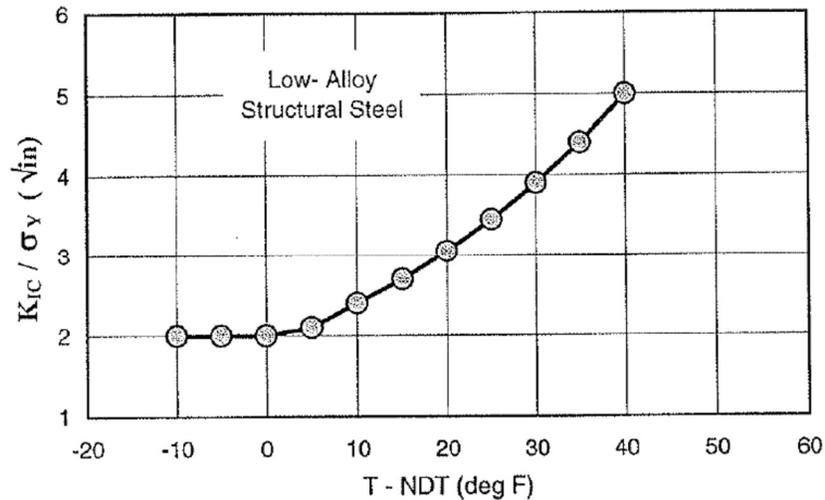


Figure 12. Variation in Fracture Toughness with Temperature

Using the above fracture toughness data ($K_{IC} = K_{IR}$), curves such as shown earlier on the Failure Analysis Diagram can be constructed for this steel using the thru-crack equation. (Fig 13)

$$\sigma = \frac{K_{IC}}{\pi\sqrt{C}}$$

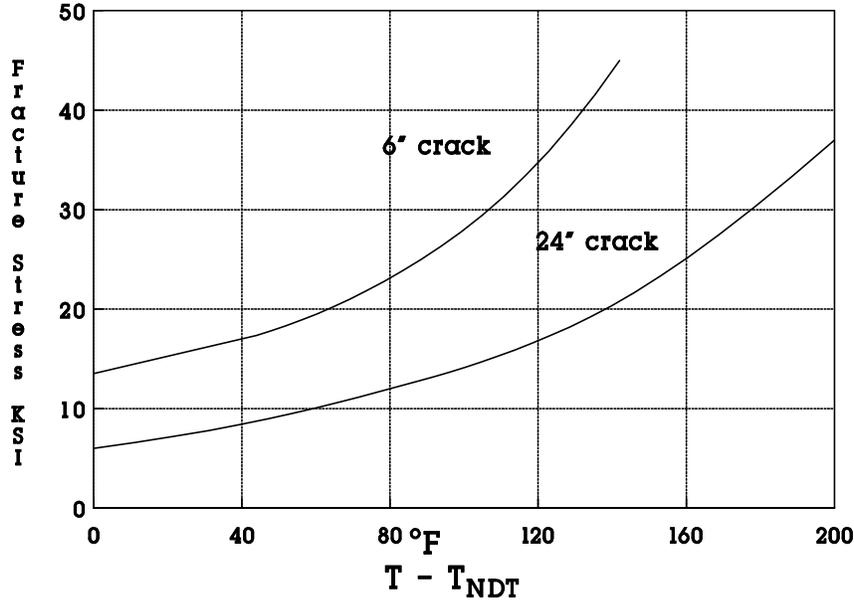


Figure 13: Fracture Toughness with Temperature

Example. A confining vessel for a nuclear reactor is designed to withstand a net pressure of 2000 psi. The outside radius is 80 inches and the wall thickness is 9 inches. The material is steel with a yield stress of 40 ksi at an operating temperature of 500°F. A hypothetical surface flaw is postulated (as part of the safety code requirements) with a depth of 1/9 the wall thickness and a length 2/3 times the thickness.

- (a) What is K_{IC} ?
- (a) If the operating temperature is 500°F, what is the required NDT?
- (b) What about at 300°F?

Solution. We have the equation

$$\sigma = \frac{K_{IC}}{\sqrt{\pi a}} \left[\frac{\phi^2 - 0.2(\sigma - \sigma_y)^2}{1.2} \right]^{1/2}$$

The values of the crack dimensions are $a = (1/9)(9) = 1$ in and $2c = (2/3)(9) = 6$. Therefore $a/c = 0.33$ and $\phi = 1.1$. The stress is

$$\sigma = \frac{Pr}{t} = \frac{(2000)(80)}{9} = 17,800(\text{p.s.i})$$

or 17.8 ksi. Solving for K_{IC} , we find $K_{IC} = 47.9$ ksi in^{1/2}. From the curve of fracture toughness (figure 13) and the intersection of a horizontal line from $\sigma = 17.8$ KSI and the 6 inch curve, we then have $T - T_{NDT} \approx 55$

- (a) for $T = 500^\circ\text{F}$ we require $T_{NDT} = 445^\circ\text{F}$ (or lower).

(b) for $T = 300^{\circ}F$ we require $T_{NDT} = 2450^{\circ}F$ (or lower).

Initially T_{NDT} may be low (say $-50^{\circ}F$), but with radiation embrittlement it can increase to $250 - 350^{\circ}F$ over a 20 year period. If it has increased to $280^{\circ}F$ and a sudden cooling of the reactor water to $300^{\circ}F$ occurred, brittle fracture could result. This problem is very obviously one of great concern. This phenomenon is known as "pressurized thermal shock".

11.6 Stress Corrosion Cracking (SCC)

Stress corrosion cracking refers to the fracture of metals and alloys under tensile stress in a corrosive environment when the applied stress is below the yield stress for the material. The fracture does not occur immediately, but instead occurs after a finite amount of time. During this time localized corrosion, caused by a specific combination of electrolyte and metal, results in the formation of cracks. Continued corrosion and solid formation within the crack combine to, in effect, increase the applied tensile load. As shown in figure 14, the allowable stress decreases the longer the particular environment exists. The phenomenon can be expressed in terms of modern fracture mechanics in terms as follows: A "stress intensity" factor is defined for a thru crack as

$$K_q = \sigma\pi(c)^{1/2}$$

where σ is the applied stress and c is one-half the width of the crack. We have already seen that fracture will occur when $K = K_c$.

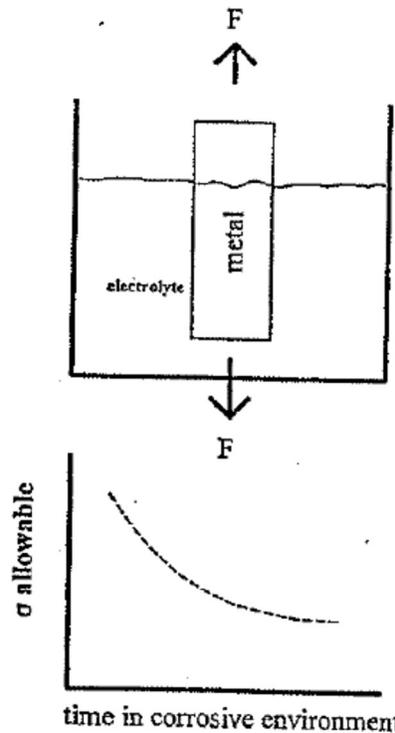


Figure 14: Stress Corrosion Cracking

Experiments have shown that with certain high-strength steels, as well as stainless steels and aluminum alloys, crack growth can occur when the material is under stress and in a corrosive environment. In this case, the stress intensity, which is a function of flaw geometry, can increase with time until it reaches the critical K_c value and failure occurs. If the stress intensity K is initially less than a critical value K_{SCC} for the material, no crack growth will occur. This is illustrated in figure 15.

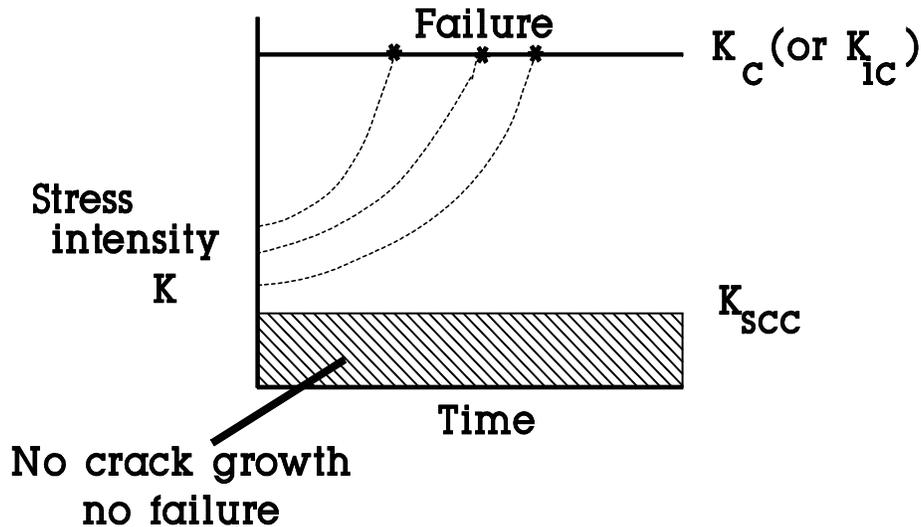


Figure 15: Stress Corrosion Cracking

11.7 Non-Destructive Testing

Nondestructive testing (NDT) methods are inspections for material defects, such as the cracks discussed in this chapter.

External (Surface) Inspection Techniques

The three most commonly used external (surface) inspection techniques currently in use are the Visual Test, Dye Penetrant Test, and Magnetic Particle Test.

- **Visual Testing (VT)** should be done during all phases of maintenance. It can usually be performed quickly and easily and at virtually no cost. Sometimes photographs are made as a permanent record. Visual inspections only allow the inspector to examine the surface of a material.
- **Dye Penetrant Testing (PT)** uses dyes in order to make surface flaws visible to the naked eye. It can be used as a field inspection for glass, metal, castings, forgings, and welds. The technique is simple and inexpensive and is shown schematically at Figure 5.10. Only surface defects may be detected, and great care must be taken to ensure cleanliness.

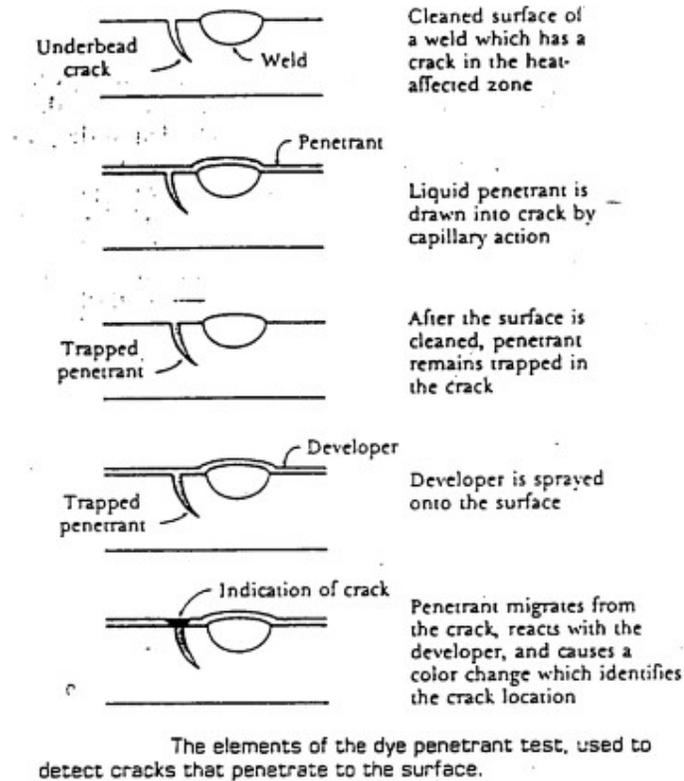


Fig 16: Dye Penetrant Testing

- Magnetic Particle Testing (MT)** is only used on ferromagnetic materials. This method involves covering the test area with iron filings and using magnetic fields to align the filings with defects. Figure 17 shows the deformation of a magnetic field by the presence of a defect. Magnetic particle tests may detect surface and shallow subsurface flaws, and weld defects. It is simple and inexpensive to perform, however a power source is required to apply the magnetic field.

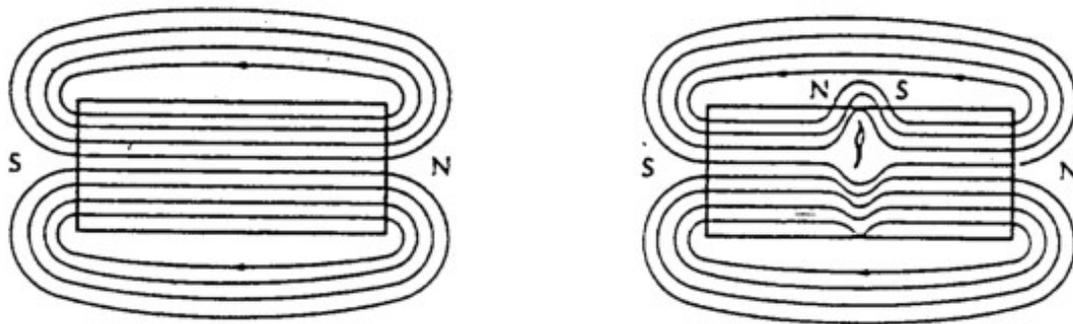


Figure 17 Magnetic Particle Testing

Internal (Sub-surface) Inspection Techniques

The three most common internal (subsurface) techniques are the Ultrasonic Testing, Radiographic Testing, and Eddy Current Testing.

- **Radiographic Testing (RT)** is accomplished by exposing photographic film to gamma or x-ray sources. This type of testing detects a wide variety of internal flaws of thin or thick sections and provides a permanent record. These methods of testing require trained technicians and present radiation hazards during testing.
- **Ultrasonic Testing (UT)** utilizes a transducer to send sound waves through a material. It may be used on all metals and nonmetallic materials. UT is an excellent technique for detecting deep flaws in tubing, rods, brazed and adhesive-jointed joints. The equipment is portable, sensitive and accurate. Interpretation of the results requires a trained technician. Figure 18 shows an ultrasonic transducer configuration.

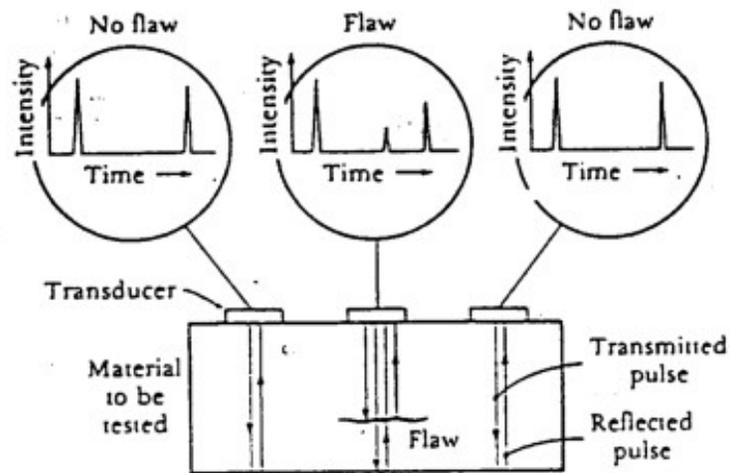


Figure 18 Ultrasonic Testing

- **Eddy Current Testing** involves the creation of a magnetic field in a specimen and reading field variations on an oscilloscope. It is used for the measurement of wall thicknesses and the detection of longitudinal seams and cracks in tubing. Test results may be affected by a wide variety of external factors. This method can only be used on very conductive materials, and is only good for a limited penetration depth. Once very common, it is being replaced by the increasing usage of ultrasonic testing.

Figure 19 summarizes these non-destructive testing techniques.

TEST	MEASURES	USED FOR	ADVANTAGES	LIMITATIONS
Visual Test (VT)	<ol style="list-style-type: none"> 1. Finish 2. Surface Defects 	ALL	<ol style="list-style-type: none"> 1. Cheap 2. Easy, no Equipment Required 	<ol style="list-style-type: none"> 1. Only for surface defects 2. No quantitative result.
ROCKWELL HARDNESS	<ol style="list-style-type: none"> 1. Hardness (Strength) 	<ol style="list-style-type: none"> 1. Testing the strength of metals 	<ol style="list-style-type: none"> 1. Non-destructive 	<ol style="list-style-type: none"> 1. Not exact value of Strength
RADIOGRAPHIC (RT)	<ol style="list-style-type: none"> 1. Internal Defects 2. Density Variations 	<ol style="list-style-type: none"> 1. Welds 2. Forgings 3. Castings 	<ol style="list-style-type: none"> 1. Gives Permanent record. 2. Great Penetration 3. Good on most geometries 4. Portable 5. Sensitive to density variations 	<ol style="list-style-type: none"> 1. Costly 2. Radiation Hazard 3. Need highly skilled operators
DYE PENETRANT (DT)	<ol style="list-style-type: none"> 1. Surface Defects 2. Porosities open to the surface 	<ol style="list-style-type: none"> 1. Welds 2. Forgings 3. Castings 	<ol style="list-style-type: none"> 1. Low COST 2. Portable 3. Easily interpreted 	<ol style="list-style-type: none"> 1. Surface defects only 2. Must clean surface before and after test
MAGNETIC PARTICLE (MT)	<ol style="list-style-type: none"> 1. Surface, shallow subsurface flaws 2. Cracks and Porosities 	<ol style="list-style-type: none"> 1. Ferrous Materials 2. Forgings and Castings 	<ol style="list-style-type: none"> 1. Can locate very tight cracks which might not see with Dye 2. Low Cost 3. Fairly portable 4. Subsurface capability 	<ol style="list-style-type: none"> 1. Alignment of magnetic field is critical 2. Must demagnetize after the test 3. Must clean magnetic dust after test 4. Surface coating masks results.
EDDY CURRENT	<ol style="list-style-type: none"> 1. Surface and shallow Subsurface defects 2. Alloy content 	<ol style="list-style-type: none"> 1. Tubes 2. Wires 3. Ball Bearings 	<ol style="list-style-type: none"> 1. High Speed 2. Automated 3. Gives Permanent Record 	<ol style="list-style-type: none"> 1. Need a Conductive material 2. Requires reference standard 3. Shallow defects only 4. Standard Geometry only
ULTRASONIC (UT)	<ol style="list-style-type: none"> 1. Internal Defects 2. Material Thickness 3. Delaminations in Composites 4. Young's Modulus 	<ol style="list-style-type: none"> 1. Welds/Brazed Joints 2. Wrought Metals 3. Hull Thickness 4. In-Service Parts 	<ol style="list-style-type: none"> 1. Most sensitive to Cracks 2. Results known Immediately 3. Permanent record 4. Portable 5. High Penetration 	<ol style="list-style-type: none"> 1. Only on limited Geometries 2. Need Trained Operators

Fig. 19 Comparison of Non-Destructive Testing Techniques