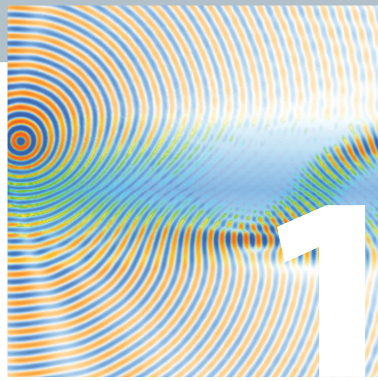

How Sound Propagates

Sound takes place when bodies strike the air, ... by its being moved in a corresponding manner; the air being contracted and expanded and overtaken, and again struck by the impulses of the breath and the strings, for when air falls upon and strikes the air which is next to it, the air is carried forward with an impetus, and that which is contiguous to the first is carried onward; so that the same voice spreads every way as far as the motion of the air takes place.

—Aristotle (384–322 BCE), *Treatise on Sound and Hearing*

More than two thousand years ago, Aristotle correctly declared that sound consists of the propagation of air pressure variations.

Even to the casual observer, sound is plainly revealed to be a consequence of vibrating or pulsating objects in contact with air. Surfaces feel a force from all the molecules colliding with them; every molecule bouncing off the surface gives it a tiny shove. A bounce is a change of velocity and thus an *acceleration*, imparting a force F on the molecule (and an equal and opposite force acting on the surface) according to Sir Isaac Newton's law $F = ma$, where m is the mass of the accelerated molecule, and a is the acceleration.

Fluctuations of pressure above (*condensations*) and below (*rarefactions*) the average pressure, arriving at the surface as sound, cause a very small increase or decrease in the number of collisions per second, and a corresponding tiny but measurable change of force on the surface. These fluctuations above and below the ambient pressure are called the *pressure amplitude* δP, where the total pressure is $P = P_0 + \delta P$, and P_0 is the ambient pressure. Usually only the amplitudes matter to us; it is changes in pressure that we hear, not the ambient pressure. We (and other animals) however are *spectacularly* sensitive to these changes; a pressure fluctuation of just a few parts in a billion (a few billionths of an atmosphere) is enough for us to hear if it happens fast enough.

As small as it is, the tympanum is huge on the molecular scale. There are so many molecules colliding with it every millisecond (roughly 10^{23}—that's 1 followed by 23 zeros) that they average out and give a *nearly* steady pressure, amounting to about 14 lb of force on every square inch. Air pressure is usually measured in kilopascals (1 kPa = 0.145 pounds per square inch, or psi). Sea-level air pressure is about 100 kPa, or 14.5 psi. The tympanum membrane, which separates the middle and outer ear, normally has equal air pressure on both sides, so there is no net force on it, except for tiny fluctuations.

Aristotle could not have known that air is a seething mass of molecules crashing into one another. More than a billion collisions are suffered by every molecule every second at sea level and room temperature. In spite of all the collisions, air is mostly empty space: the molecules occupy only about one part in 5000 of the available volume. Think of 10 bumper cars in an area the size of a football field. You might think that this was a relatively safe, low density of cars—unless each car was traveling at thousands of kilometers per hour. There would be many collisions every second. Between collisions, molecules speed along a straight path at typically half a kilometer per second, managing to travel only a tenth of the length of a typical bacterium before suffering another collision.

The density and speed of air molecules are in this way sufficient to explain atmospheric air pressure and the speed of sound. Individually, the air molecules (mostly diatomic nitrogen and oxygen) act like drunken messengers flying and colliding every which way. Nonetheless, these collisions can collectively communicate even slight fluctuations in pressure to neighboring collections of molecules, which in turn pass them on to their neighbors, leading to sound propagation. Air molecules are usually not traveling directly along the path of the sound wave; the information that there is higher or lower pressure somewhere propagates no faster than the average speed of molecules along a given direction.

The typical 500 meter/second (m/s) molecule is traveling either in the wrong direction or only 300 to 400 m/s along the direction of propagation of the sound. Thus the effective speed with which the morass of molecules communicates pressure variations is less than their average speed of 500 m/s. The measured speed of sound in air is about 343 m/s at room temperature.

The "seething mass of molecules" picture explains why the speed of sound is insensitive to pressure, since pressure hardly affects the speed of individual molecules. They crash into each other more often at high pressure, but between collisions they travel at a speed that depends only on the temperature, not the pressure. The speed of sound on Mount Everest is nearly the same as at sea level, if the temperatures are the same.

The average speed of molecules is proportional to the square root of the temperature, and inversely proportional to the square root of the mass of the molecules in the gas.

"Helium voice," the Donald Duck–like sound when someone speaking has just inhaled a puff of helium, is the result of the much higher speed of sound in helium than in air. Helium has a mass of four atomic units; air has an average mass of about 29 atomic units and $\sqrt{29/4} \approx 2.7$. The speed of sound in helium, 972 m/s, is about 2.8 times that of air, at 343 m/s. Another harmless gas (except that like helium, it displaces oxygen and can be lethal if breathed for more than a short time), sulfur hexaflouride, SF_6, is much heavier at 146 atomic units and should have a speed of $343 \times \sqrt{29/146} = 153$ m/s; the measured value is 150, less than half the speed of sound in air. "SF_6 voice" is even more astonishing in its effect than helium voice, and in the opposite direction. (However, the nature of and reasons for the changes in the sound of the voice using helium and SF_6 will be explained in section 17.9. In spite of impressions, the gases do not change the pitch of the voice!)

The energy needed to make audible sound is very small. You can shout for a year, and the energy produced *that winds up as sound* would not be enough to boil a cup of water. A full orchestra playing loudly produces only about enough sound energy to power a weak lightbulb. An orchestral crescendo might bathe a listener in sound pressure fluctuations of about 1 pascal (1 Pa). Sea-level air pressure is 100,000 Pa, so the crescendo loud enough to damage your hearing, if it lingered too long, is varying the pressure by just 0.001%. Clearly, a very delicate detection system is at work. We will find in chapter 21 that human hearing depends on a few thousand *single-molecule* links between cochlear hair cells.

At the extreme—loud sound near the threshold of pain—the air pressure variations are over a million times bigger than the threshold of hearing, or about a 0.03% pressure variation, 30 Pa or so. This still seems small, and yet is almost immediately damaging! This sound level corresponds to a power arriving at the ear 10,000,000,000,000 (10^{13}) times larger than that which produces the softest sound we can hear. (The power increases as the square of the pressure variations.) The dynamic range of our hearing is truly remarkable.

Why should you buy a 600-watt (W) amplifier for your loudspeakers if a full orchestra normally produces just a watt of power, 40 or 50 W at the loudest? The answer is that to reproduce sound, rather large forces must be exerted on the speaker cone to get it to vibrate in a prescribed way. The conversion efficiency from motion of a loudspeaker cone to sound is very low. The cones are moved with electric currents in coils near magnets, wasting considerable energy. Imagine all the effort you would expend waving your hands back and forth 1000 times. Only a tiny fraction of that energy would go into pushing air around; most of the energy expended would go into working against yourself, so to speak: internal friction, stopping your arms with one set of muscles after starting them swinging with another, working against gravity, and so on. So it is with a loudspeaker. For that matter, musicians can work up a sweat playing an instrument, all to produce well under a tenth of a watt of sound.

1.1 Push and Pushback: Impedance

We need to develop a better intuitive foundation for sound propagation. The "drunken messenger" picture explains the speed of sound but applies on the molecular scale, too small to give a good sense of wave phenomena such as reflection, diffraction, refraction, and so on. For example, much of the sound traveling down a tube reflects from its open end, reversing direction rather than exiting to freedom. Why doesn't the sound just leave the tube? Why is the reflected wave a *rarefaction* (pressure low relative to ambient) if the incident wave approaching the end of the tube was a *compression* (pressure high relative to ambient)? Why does sound of high-enough *frequency* (the frequency is the number of wave crests traveling by per second), on the other hand, mostly escape the tube without reflecting? There are not many references that provide a foundation for a comprehensive understanding of these sorts of phenomena; those written for engineers and physicists all too often derive equations and formulas but skimp on the intuition.

Imagine dividing air into small cells. Each cell is large on the molecular scale; they are packed one next to the other. The size of the cell is determined by the wavelength of the sound involved (there needs to be at least several cells per wavelength) and the details of any obstacles, sound sources, and so on. If we can understand how the cells communicate with each other, are pushed by and then push back on neighboring cells, we can understand propagation, reflection, diffraction, and even refraction of sound. This is our first glance at a powerful engineer's trick, wherein the properties of complex objects are lumped into a few well-chosen summarizing properties. These have vastly less information than the original system, but enough to carry the essential physics, and lead more easily to the correct conclusions.

To understand impedance in air, we begin by considering solid elastic bodies, such as pucks on an air hockey table or coins on a slick surface. We need to understand such things in any case, because usually before air is set in motion, something more massive, like a string or a block of wood, is set in motion. Each puck or coin is a lumped object—we ignore the details of atomic or molecular structure inside, but keep essentials such as size, density, and elasticity, just as we will for air cells when we return to them. The essentials are used to build a theory of what happens when adjacent lumps interact. You may have noticed, for example, that in a head-on collision between two pennies, one initially at rest, the moving penny stops dead in its tracks, and the second one picks up where the first one left off. (The demonstration does not work well with quarters or coins having serrated, gearlike edges colliding with other such coins. Presumably the serrations cause a rather nonideal collision, gnashing of the gears, chattering, and so on.)

Complete transfer of momentum does not happen when a nickel collides with a penny at rest, nor when a penny collides with a nickel at rest. The energy of the first, moving coin is only partially transferred to the second. If we make a line of coins, each coin becomes an agent of transfer of energy from left to right, if the first coin was traveling in that direction. A coin that is much heavier or lighter than its neighbors will impede the transfer of energy. Two nickels are *impedance matched*; the stationary nickel gives as good as it gets, stopping the moving nickel dead. A penny and a nickel are impedance mismatched; a penny does not exert as much force back on the colliding nickel as another nickel would and does not decelerate the nickel all the way to zero speed. The nickel continues on its way, albeit more slowly. Only in the case of equal masses does the energy get completely transferred from one coin to the other; this is clear since for the head-on nickel-penny and penny-nickel collisions, both coins remain in motion and that movement carries energy.

If you line up 5 or 6 pennies perfectly on a slick surface and hit the end of the row head-on with another penny, you will notice the row stays intact, with the projectile penny adding to the row and the last penny popping off at the opposite end. The impedance matching works all the way down the penny chain, each penny for an instant carrying the momentum, giving as good as it got on its left, and then almost instantly giving and getting forces on its right that stop it cold and give the momentum to the next penny. Put a nickel in the chain of pennies and the first penny will rebound from the row; the last will still pop off the end but with less energy than before. All the energy of the first coin is not transferred down the chain; rather, part of the energy has been reflected and part transmitted, *because of the impedance mismatch, which can be blamed on the interloping nickel.* The situation is depicted in figure 1.1.

The impedance of the untethered coins is proportional to their mass. Two untethered objects of equal mass, therefore, indeed have the same impedance. The bigger the impedance mismatch, the more energy is reflected and the less transmitted. The formula for the fraction of energy

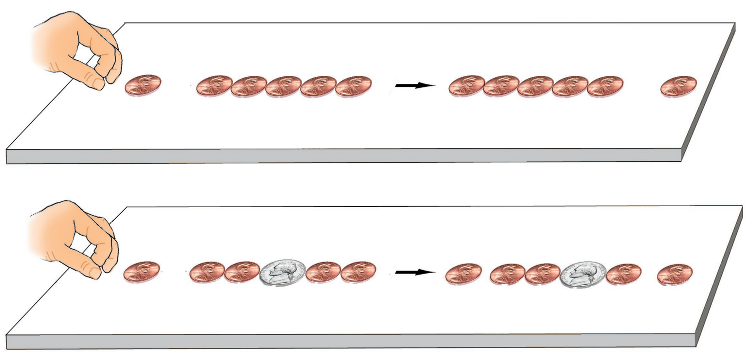

Figure 1.1

In the top row, a penny collides head-on with a row of five pennies, resulting in the expulsion of the last penny in the row with the same speed as the first penny had. The masses are all the same and the chain of pennies is impedance matched, resulting in 100% transfer of the energy from the first penny to the last, except for friction. In the bottom row, the presence of the nickel replacing one of the pennies causes a mismatched impedance, with some of the energy reflected back toward the first penny, causing it to rebound; only part of the energy reaches the last penny.

R that the moving mass m_1 retains in a head-on collision with a stationary mass m_2 is

$$R = \frac{(m_1 - m_2)^2}{(m_1 + m_2)^2}. \tag{1.1}$$

If one coin weighs one-tenth as much as the first, say, $m_1 = 1$, $m_2 = 10$, R will be $9^2/11^2 = 81/121$, which means 67% of the energy gets reflected on one bounce, and 33% transmitted.

What Is Impedance, Really?

Roughly speaking, impedance, which we symbolize with the letter Z, measures the response of a body to a force—in fact, the force applied divided by the velocity attained ($Z \sim$ force $\div$ velocity). A heavy object moves slower than a light one after the same force is applied starting at rest, so impedance is high for a heavy object, low for a light one. This is still a rough definition, since in the measurement of Z, the force is taken to vary sinusoidally (see chapter 3), and the velocity, while also sinusoidal, may lag or lead the force. We will consider these complications later.

With this notion of Z (force applied $\div$ velocity attained), it is possible to see why matched impedance leads to complete energy transfer between two bodies. According to one of Newton's laws, they experience equal and opposite force as they collide or interact, and what velocity is lost by one is gained by the other—just the ticket if you want to transfer energy from one place to another, or from one thing to another. One coin stops and the other takes off with the same velocity.

The utility of impedance is to help determine the transfer of energy between bodies. Matched impedance means efficient energy transfer; unequal impedances mean rejection or reflection of energy. Ideally, impedance can be determined for any part of an object, such as a block of metal or a section of pipe with air in it. If two such objects are joined somehow, an impedance mismatch (if any) can be calculated, and the *transduction* (transfer) of energy from one part to another can then be determined.

As an example, suppose two strings of different density are tied together. We will see in chapter 8 that waves travel down a uniform string quite readily, with a velocity $c = \sqrt{T/\rho}$, where T is the *tension* (a force) along the string, and ρ is the *density* (mass per unit length) of the string. The two parts tied together have the same tension, since tension is communicated all along the string, but they have different density, and thus different wave speeds c. They also have different impedances. The impedance of transverse oscillations of a stretched string is

$$Z = \sqrt{T\rho}. \tag{1.2}$$

Given the densities ρ_1 and ρ_2 of the two string segments, we can easily calculate the reflection and transmission of energy at their junction using formulas 1.3 and 1.4 given below.

Antireflection Strategies

Suppose we insert a third coin between two mismatched coins, one more massive than the other. The middle coin should be of some intermediate mass, to make the mismatches of adjacent coins less severe. It is not difficult to show that taking the mass of the middle coin to be the geometric mean of the two original coins (that is, $m = \sqrt{m_1 m_2}$) is optimal. The transmission with the intermediate coin in place in the 1:10 impedance mismatch considered earlier then works out to 53% from the first to the last coin; an improvement over the previous 33%. We would do even better with more intermediate coins selected to further reduce the adjacent impedance mismatches.

Abrupt changes in impedance at a boundary between two objects or regions lead to low transmission of energy across the boundary. Like the nickel in a line of pennies, regions with different impedance push back too much or too little. Suppose we have a system of one impedance Z_1 on the left side connected to a second system on the right with a different impedance Z_2. The sudden change of impedance at the interface causes a fraction of energy **R** to be reflected:

$$\mathbf{R} = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}. \tag{1.3}$$

Thus equation 1.1 generalizes to more general types of impedance, including (as we shall see) restoring force and friction. The *transmitted* energy is

$$\mathbf{T} = 4\frac{(Z_1 Z_2)}{(Z_1 + Z_2)^2}, \tag{1.4}$$

and the reflected and transmitted fractions sum to one: $\mathbf{R} + \mathbf{T} = 1$—that is, what is not transmitted is reflected.[1]

Impedance matching plays a role in many domains. In the preceding example, the coins were a "medium" for the propagation of the translational energy possessed by the first coin. Light is similar: it propagates nicely through transparent media, such as air and glass, but these do not have the same impedance. The impedance (called *refractive index* in the case of light) has a mismatch passing from air to glass, with the result that some

[1]The impedances are in fact complex numbers, so we have $\mathbf{R} = (|Z_1 - Z_2|^2/|Z_1 + Z_2|^2)$ and $\mathbf{T} = 4[\mathrm{Re}(Z_1^* Z_2)/|Z_1 + Z_2|^2]$, where Re denotes the real part of the variables within the parenthesis, and $|\cdots|^2$ is the absolute value squared of $\cdots$.

light will reflect at the interface, whether it is coming from air to glass or vice versa. If a coating can be found with intermediate impedance, it can break up the impedance mismatch into two smaller steps, with the result that less light will be reflected and more transmitted. This is the principle of antireflection-coated eyeglasses and camera lenses. The coating works better for some colors (wavelengths) than others; this explains the color sheen often seen on coated optics.

As an example of the importance of impedance to sound and music, consider a violin. The body of a violin is much heavier and stiffer than a string and has a much higher impedance. Both impedances vary with frequency too. The body needs to tap into the energy of the string in order to make sound. (Vibrating strings by themselves are almost silent—this will be made clear in the following chapters; see especially the discussion of dipole sources—for example, section 7.7). Hypothetically the string could be attached directly to the body, but there are several problems with this (see figure 1.2). The directly connected string may not set the correct body vibrations into play. Worse, there is a large impedance mismatch between string and body, preventing the string from imparting enough of its energy to the violin. (Note: We don't want the transfer of energy from string to body to be *too* efficient either, lest the string dump its energy too fast.)

Figure 1.2
What would this sound like? A string is attached directly to a violin body at one spot (no bridge) and to a rigid wall at another. It is bowed in the usual way.

Impedance and the Violin

Air has a refractive index n_{air} of about 1, and glass can be $n_{glass} = 1.5$ or so. The refractive index is essentially impedance; the formula for the fraction of light reflected is

$$R = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}. \quad (1.5)$$

This is a 4% reflection of light for air–glass, for each surface, and there are always at least two surfaces and sometimes many more, as in expensive camera lenses. By adding an optimal single coating, with the geometric mean refractive index $\sqrt{n_{air} n_{glass}}$, we can get this down to a 2% reflection. Multiple coatings can do even better.

Can something be inserted between string and violin body to lessen the impedance mismatch, thus allowing the energy to take two smaller steps, instead of one large one? While we are at it, can we sweeten the sound by modulating the impedance (and ultimately the loudness of the instrument) according to frequency? The answer is yes: this is the job of the bridge, as we discuss in chapter 18. The bridge is the "intermediate coin" that mediates the transfer of energy from string to body. Its impedance is cleverly tuned by choice of shape, size, and material to depend in a certain way on the frequency of vibration.

Bullwhip—The High Art of Impedance Matching

The bullwhip is a spectacular example of impedance matching (figure 1.3). If most of the energy from the relatively heavy handle region can somehow be efficiently transferred to a light string ("popper") at the other end, the popper will wind up moving very fast. Sudden impedance mismatches along the whip would reflect energy, so the bullwhip is gradually tapered and also carefully constructed so as to have no abrupt changes in density or stiffness. The energy of a moving mass m due to its motion is $E = \frac{1}{2}mv^2$, where v is its velocity. A reasonable estimate is that the popper weighs 1/400th as much per centimeter of length as does the handle end. The energy per centimeter if the handle region weighs M kilograms per centimeter is $E = 1/2MV^2$, where M is the mass of a centimeter near the handle end, and V its initial velocity. If this gets transferred to the popper, then the same energy is now written $E = 1/2mv^2$, where m is the mass per centimeter of the popper, and v is the velocity of the popper. The ratio of the two velocities is

$$\frac{v}{V} = \sqrt{\frac{M}{m}} = 20 \qquad (1.6)$$

in this case. A factor of 20 does not sound huge, until you realize it is easy to get the handle moving at 40 miles an hour (a fast baseball pitch is 100 miles per hour), and 20 times that is 800 miles per hour, or faster than the speed of sound at 770 miles per hour! The popper thus goes *supersonic* (faster than the speed of sound). A supersonic object traveling through the air creates a shock wave, a very sharp pressure pulse. (More on supersonics and shock waves in section 7.9.) The pulse itself travels through the air at the speed of sound, but when it reaches the ear, it is heard as a loud bang.

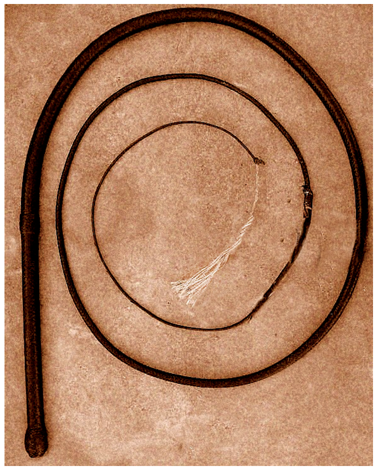

Figure 1.3

An Australian bullwhip can achieve supersonic speeds at the whip end, resulting in a loud crack heard some distance away. Courtesy Cgoodwin.

Impedance Mismatches Are Not Always Bad

One does not always want to maximize energy flow across junctions between two parts of a system. We *need* the impedance mismatch at the bell end of a trumpet or clarinet to cause reflections and define its resonance frequencies. Impedance mismatches are carefully controlled to achieve desirable timbre in wind and string instruments. For string instruments, large mismatches are required at the points between which strings are stretched, lest the vibrations drain away too rapidly, rendering the string frequencies ill-defined. The infamous wolf note of cellos is a breakdown of this requirement (see section 18.7)—a near impedance matching where none was wanted.

Impedance of Masses and Springs Together

Untethered coins on a table move along without hindrance (except for friction, which we have neglected so far) but many objects are tied down and experience a restoring force pulling them back if they are displaced. The concept of impedance applies, but now impedance can be high owing not only to large mass but also to large stiffness because of a spring, which also tends to keep speed low. A mass and spring can combine to make an oscillator that vibrates at a certain natural frequency; if you push back and forth at that frequency, the impedance is low even if the mass is large and the spring is strong, because the oscillator gets moving very fast.

Three universal properties of matter figure into impedance: (1) *Mass* is responsible for resistance to *acceleration*, as is encoded in Newton's second law of motion $F = ma$ (force = mass × acceleration). For a given force, acceleration and mass are inversely proportional. (2) *Stiffness* is responsible for resistance to being stretched or compressed, as encoded in the spring equation $F = -kx$, where F is the force, k is the spring constant, and x is the displacement. (3) The third universal property is *friction*. We are deferring that topic for the moment; see section 10.6.

If the force is applied slowly, acceleration is weak. The force is then usually governed by compressibility or springiness, which therefore gives *stiffness-dominated impedance*. If a force is applied suddenly, the object hardly has time to move and sense its stiffness, but the mass of the object is felt immediately; the impedance is mass dominated.

Defining and Measuring Impedance

We measure impedance by applying a back-and-forth, sinusoidal forcing. (The sinusoid is the subject of chapter 3.) The impedance will depend on the frequency of this forcing. If the *period* (time interval between repetition of the periodic force) of the forcing is very short (high frequency), then the force is changing suddenly; not much movement of the object takes place because such a short time elapses between reversals of the force. The impedance will tend to be mass dominated. If the frequency is low and the forcing period is very long, then the force is being applied slowly; the impedance will tend to be stiffness dominated. The object or matter in question is forced according to $F(t) = F \sin(2\pi ft)$; this periodically pushes right and left with frequency f. The sine function never gets bigger than 1, so the maximum force is F.

The object or matter being forced sinusoidally will temporarily build up speed in one direction and then slow down, stop, and reverse direction, building up speed in the opposite direction. Reaching high speed suggests a large response to the forcing, which in turn implies that the object presents low resistance—that is, low impedance, to energy *at the forcing frequency* f. The frequency-dependent impedance $Z(f)$ is defined as the ratio of the maximum force F to the maximum speed $u(f)$ reached at that frequency f:

$$Z(f) = \frac{F}{u(f)}. \tag{1.7}$$

The higher the speed $u(f)$ attained, the lower the impedance. This definition still ignores the phase lag or lead of the velocity relative to the force. The impedance $Z(f)$ used by engineers is a complex number—that is, it contains the imaginary number $\sqrt{-1}$; however, we will suppress that fact. (The information we throw away by doing this is the phase [see chapter 3] of the velocity attained relative to the forcing. We will discuss the phase quite thoroughly, but will not write it in terms of complex numbers.) Impedance is a measure of the ratio of the (sinusoidal) force applied to the speed attained. If we apply a large force and don't get much speed out of our efforts, the impedance is high. If for the same force, the point where the force is being applied reaches a high velocity, the impedance is low. It is important to remember that in our simplified version of impedance, the force is calculated as the maximum force at the point of application, and u is the maximum speed attained by that point.

To measure impedance, we can control the force and then measure the resulting speed—that is, control the numerator and measure the denominator in expression 1.7. Or we can control the speed of the point of application, and then measure the force that is needed to maintain that speed—that is, we can control the denominator and measure the numerator in equation 1.7. If the force or the velocity is controlled at the same spot on the object, the same value for the impedance is obtained either way. Extended objects will have different impedances depending on where the force is applied.

1.2 Impedance of Air

The idea of "push and pushback" and impedance can now be made more precise for air. Air has mass and is springy—so there ought to be a way to connect air to the impedance ideas we just discussed. Again we arbitrarily divide up a body of air into cells. The cell walls are purely mathematical—completely elastic and having no mass of their own. They do not exert any force or pressure of their own, but rather just follow along with the adjacent air. This division into cells does no harm, yet it helps our thinking. Each cell has mass and springiness. It is in contact with other cells with their own mass and springiness. Taking the cells to be cubic, if we push on one side of a cell, it will tend to bulge out on five other sides.

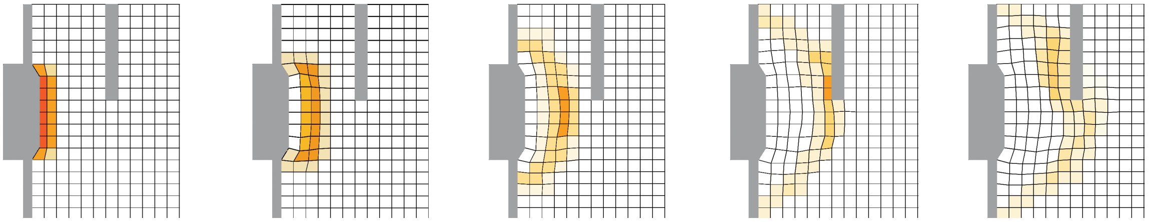

Figure 1.4
A piston initiates a pressure pulse in the cellular picture of sound propagation. Propagation, reflection, and diffraction are all represented.

The restoring force that any given cell presents upon being pushed on one side depends on how much pushback it receives when it tries to bulge out on the other sides. If one side of the cell is up against a rigid wall, the pushback from pushing on any other side will be higher, since one side can't move at all. Thus the presence of the wall causes an impedance change.

The impedance of a cell of air has three components: a component due to the mass of the air inside, a component due to the restoring force or springiness of the air, and a component due to friction, which we can safely ignore if the air is far enough from surfaces. In analogy with our line of pennies, cells of air are stacked next to each other, in three dimensions rather than one. Normally, each cell of air is just like the ones adjacent, which strongly suggests that air is impedance matched with itself and will efficiently transmit propagating sound.

Let's see how this works to explain the propagation of sound. Figure 1.4 shows a sequence of five snapshots in the evolution of a cell system with walls and a piston present. On the left, a piston has just pushed into the area, causing a region of high pressure next to its surface. Each cell contains the same quantity of air, so smaller cells are higher pressure. The piston holds its place, and the pressure wave begins propagating by the "shove and be shoved" principle. The color shows the pressure, and the distortions of the walls of the air cells are shown. A half-wall mid-chamber intersects the wave, and in the last frame we see reflection and diffraction from the wall well under-way. The cells just next to the piston are compressed initially, but they shove their neighbors and return to normal pressure. The domino effect continues as the wave propagates.

How big do the cells need to be? There is no single answer to this question, because a few smaller cells can often be replaced by one bigger cell, but there is a limit: the cells need to be much smaller than the shortest important sound wavelength, so that the information that they are being pushed on one side travels to the other sides in a time much shorter than a period of the sound. Usually a few centimeters or, at worst, a few millimeters on a side (giant on the scale of the distance between atoms and molecules in air) will suffice. In free space, they can be about a tenth of the smallest wavelength present, or even larger. But there may be solid objects or density changes on a much smaller scale than the wavelength, which rudely interrupt the wave. If their effect is to be included accurately, especially if the listener is nearby, smaller cells need to be used near such objects.

If a cell pushes back too hard (higher impedance than its neighbor), then the neighbor doing the pushing will recoil, pushing back on *its* neighbor on the opposite side, causing a positive pressure pulse to propagate backward—a *reflection*. If the adjacent cell, on the other hand, pushes back too feebly (lower impedance than its neighbor), then the pushing neighbor will keep moving toward the weak neighbor, ultimately *pulling* on *its* neighbor on the opposite side. That neighbor pulls in turn on *its* neighbor on the opposite side, and so on. A *rarefaction* is propagating back toward the source. A positive pressure fluctuation will thus partially reflect back as a negative one if it meets reduced impedance. If the adjacent cells are impedance matched, each pushes back just enough so as not to reflect any of the pulse.

The impedance of water is about 3400 times larger than the impedance of air. You may have noticed that if you are underwater, it is very difficult to hear someone above water, even if he is shouting. Using formula 1.3 for the amount of energy reflected, we find that about 99.9% of the sound arriving from the air is reflected from the water surface. Sound launched within water travels quite well; if it reaches the surface, it reflects back down. Notice from formula 1.3 that the percent of energy reflected is the same, no matter which side of the interface the energy is approaching from.

Several types of impedances are used for air, depending on the situation. All of them are a ratio of a force to a velocity or, if you like, the ratio of a "push" to "flow."

Specific acoustical impedance z. The push or force is measured in fluids as pressure p—that is, force per unit area on a surface. The flow v is just the speed with which the small cell moves due to the pressure. The specific acoustical impedance is just the ratio of these two quantities:

$$z = \frac{p}{v}.$$

Again, we are glossing over the relative phase lag of the pressure versus the velocity; they may reach their maxima at different times under sinusoidal pressure variations.

If there are no surfaces or reflections of any sort, the specific acoustical impedance is an intrinsic property of the medium, given by the product of the density of the medium ρ_0 and the speed of sound in it c; $z = \rho_0 c$.

Acoustical impedance (lumped) Z. The specific acoustical impedance is determined at a single point. Sometimes a lumped impedance is better to work with. For example, when we want to determine the impedance mismatch and reflection upon a sudden change of pipe diameter, it is convenient to have a single lumped impedance for pipes of given diameter.

For this, the impedance definition is changed a little, so that all the cells across the pipe are lumped together and the velocity used is the *volume velocity*—that is, the velocity attained by the little cells times the area S of the pipe. For a pipe where the diameter is small compared to a wavelength, the velocity v as a sinusoidal wave passes by will be essentially uniform across the pipe, so the volume velocity is $U = v \times S$ and the acoustic impedance of the pipe of cross-sectional area S is

$$Z = \frac{p}{U} = \frac{\rho_0 c}{S}.$$

Thus the impedance of a pipe is inversely proportional to the area of the pipe.

In developing our "push and pushback" intuition for sound propagation, we are in fact coming very close to the way numerical computations are done. We will not go into the details of the algorithms here, but it is not difficult to imagine that a computer can be programmed to determine the result of all the pushing and shoving by air cells, including the effects of boundaries.

Keeping track of the air pressure variations everywhere, including the effect of various nearby surfaces, is an enormous task, even for twenty-first-century computers. However, by employing banks of *graphics-processing chips* (the computers within the computer that control screen display, called graphics-processing units, or GPUs), we can carry out the calculations required to simulate the generation and propagation of sound. GPUs became powerful and cheap primarily because of the demands of computer games. It will not be long before acoustical consulting firms will be providing their clients with accurate and perfectly detailed computer simulations of the sound in concert halls or other soundspaces, including the effects of curtains, statues, chairs, and people; sound absorbing surfaces of all sorts; open windows; and so on. The process of computing the sound pressure field—by following the movement of the sources of sound, the propagation of sound waves, and all the reflections, refractions, absorption, and so on that are present, turning it finally into a playable sound file—is called *auralization*.

1.3 Propagation of Sound in Pipes

Pipes make the whole issue of sound propagation much simpler, provided we confine ourselves to sounds whose wavelengths are long compared to the diameter of the pipe. Such long wavelengths propagate along the axis of the pipe but don't vary much from center to edge of the pipe, permitting a one-dimensional treatment in terms of the distance down the axis of the pipe. Pressure is given as a function of this distance and time along the pipe axis. This is much simpler than trying to work out all the variations in a three-dimensional sound field.

We suppose that such a pressure wave is traveling down the pipe. This is easily arranged in a number of ways, such as slapping an open end with a flat object. The propagation of such a pulse down a straight-walled tube is intuitive from the cellularization and impedance picture of the air in the pipe. First, since we will not be concerned with variations in pressure across the pipe, we can enlarge the cells into thin lozenges that extend across the pipe, taking on the cross section of the pipe. The pressure is taken to be constant everywhere in a given lozenge.

Each lozenge has mass m and is pushing out on its two neighbors; they push back just as hard in the quiescent state. If a disturbance arrives, a lozenge momentarily pushes on its neighbor a little harder, which now feels an unbalanced net force F as it begins to accelerate according to $F = ma$. The acceleration in turn induces a harder push on the next lozenge, and so on down the line, leading to propagation of the pulse.

We now turn to what happens when changes in the pipe are encountered by the pulse.

Reflection of Sound at a Closed End

The impedance of all the lozenge cells is the same because they are all identical in a pipe that does not change cross section. Suppose, however, the pulse meets a rigid end cap—that is, infinite impedance. The cell next to the wall pushes back on the adjacent cell very hard, since it has nowhere to go. This "over-pushback" causes the adjacent cell to recoil in the reverse direction; in turn, it pushes on its neighbor on the side away from the wall, and so on. There is thus a traveling pressure pulse that has reversed direction; it has bounced or reflected off the end cap with no loss of energy (figure 1.5). *Note that the end cap did not move at all to cause this reflection, or* echo*, of the sound.*

Reflection of Sound at an Open End

If a pipe terminates in an open end, it is much the same as a sudden very large increase in pipe diameter. We expect a sharp drop in impedance; the discontinuity will reflect sound amplitude back with the opposite sign. The

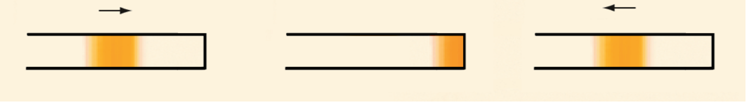

Figure 1.5
Reflection at a closed end cap in a pipe, taken directly from a Paul Falstad *Ripple* simulation. The simulation of a single half-wave, as seen here, can be set up in *Ripple* by initiating sinusoidal waves to the left of the pipes and later erasing all but one half-pulse inside the pipe before it reaches the junction.

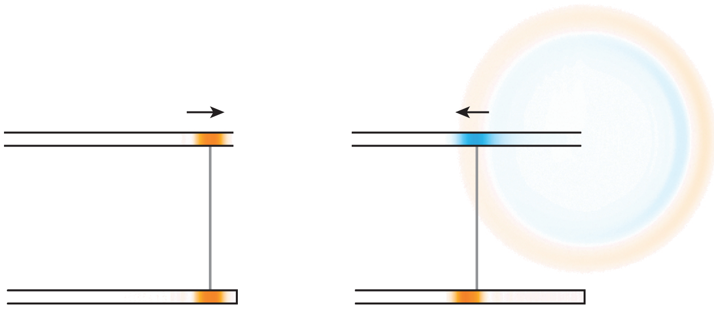

Figure 1.6

Reflection of a pressure pulse at the open end of a narrow pipe (top) and the closed end of a narrow pipe (bottom). Three significant effects are seen: First, the sign of the pulse reverses in the case of the open end, but not in the case of the closed end. Second, in the case of the open end, not much of the sound is emitted; most reflects. Third, there is a slight delay (as seen using the vertical reference line) of a pulse in the case of the open-end pipe as compared to the closed-end pipe, as if the open pipe were slightly longer. The delay is evidence of the *end correction* which makes open pipes effectively somewhat longer than their nominal physical dimensions.

air at the end of the pipe feels less pushback, overshoots, and pulls on the air behind it, initiating a rarefraction that propagates backwards.

Figures 1.6 and 1.7 show this effect quite nicely. Both figures are taken directly from *Ripple* simulations, which we can set up by drawing the pipes and sending in sinusoidal waves. The simulation is stopped, and the Erase Wave tool is used to trim the wave to lie inside the pipe and to be only half a wavelength across.

An open pipe partly reflects the wave with a change of sign. It reflects as if from a place just outside the end of the pipe, making the pipe effectively longer by about 0.6 times the diameter, for wavelengths that are large compared to the diameter.

As an interesting test of our understanding, suppose we send a pulse through a tube heading toward an open end, but this time the pulse exists outside the pipe as well. What will happen when the pulse reaches the end of the tube? The air inside the pipe has no idea that the pressure pulse exists outside until it reaches the end; as the pressure exits the pipe, instead of finding lower pressure laterally as it did before, it now finds matched higher pressure outside. There is no sudden pressure release laterally, no impedance change. The entire pulse proceeds as if nothing happened; there is no back reflection inside the pipe at all. Figure 1.7 comprises two snapshots from a *Ripple* simulation verifying this effect.

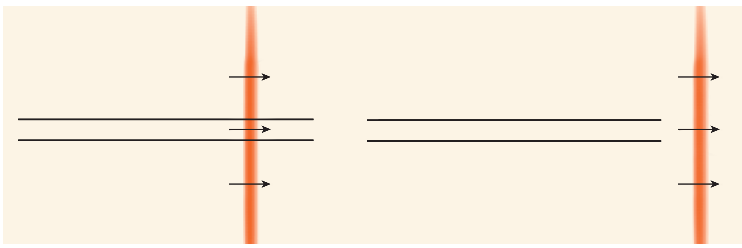

Figure 1.7

Two snapshots of a *Ripple* simulation showing a pulse propagating from left to right both inside and outside a tube. When the pulse exits the pipe, no reflection takes place.

Reflection of Sound at the Junction of Different-diameter Pipes

If the pipe changes diameter, the pulse will meet a change in impedance. Cells on the other side of the diameter change will push back too hard (if the impedance it meets is higher than its own), or too little (if the impedance it meets is lower). This will cause partial reflections of the sound at such junctions.

Earlier, we said that the impedance of air in a pipe depends on the diameter of the pipe. The bigger diameter, the lower the impedance. This makes a certain amount of sense, since a small pipe "impedes" the flow of air more than a large pipe. The impedance is again $Z_{pipe} = \frac{\rho_0 c}{S}$, where ρ_0 is the density of air, c is the speed of sound, and S is the cross-sectional area of the tube.

The physical reason for the increase of specific impedance as the pipe diameter decreases is understandable from the cellular picture. The higher specific impedance of a small pipe implies that if a small cell of air is pushed, a neighboring cell will push back harder than it would in a larger pipe. Why should this be? All the pushing and pushing back is of course communicated by the air in the pipe from cell to cell at the speed of sound. Suppose a given cell is being pushed to the right for a time τ; in free space, that push would be communicated in all directions a distance $x = c\tau$ in the time τ, where c is the velocity of sound. In the pipe, most of those directions lead to the walls of the pipe, where the pressure pulse created by the push is reflected. Some of the reflected pressure returns fast enough to the cell that was originally disturbed that it leads to an increased pushback, *while the original push is still happening and therefore in phase with the pushing*, thus increasing the impedance. "Fast enough" is in relation to the frequency of pushing. This suggests the wall needs to be within an eighth of a wavelength or so, to return in phase. Most musical instruments are

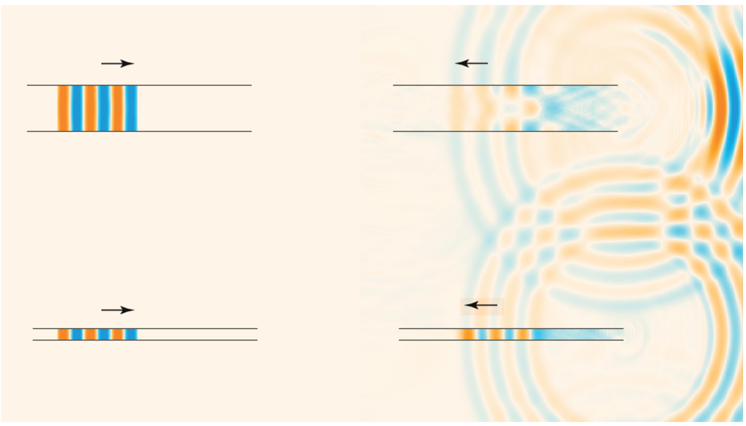

Figure 1.8

Sound of the same wavelength propagates in a narrow and a wide pipe in this *Ripple* simulation. It escapes more readily from the wide pipe, which can be seen by inspecting the intensity of the reflected waves in the right pair of panels. This can be justified using the cellular picture and impedance arguments, as explained in the text.

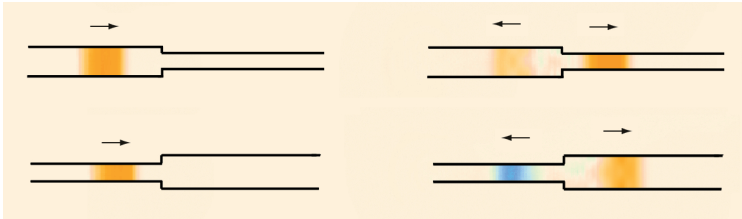

Figure 1.9

Reflection at a discontinuity in pipe diameter, taken directly from a Falstad *Ripple* simulation. (Top) A compression pulse traveling to the right encounters a smaller pipe, causing a compression reflection (same sign as the incident wave) and partial transmission of the compression pulse. (Bottom) A pulse of higher pressure (compression) traveling to the right encounters a larger pipe, causing the reflection of a rarefaction pulse (opposite sign from the incident pulse) and partial transmission of the compression pulse.

operating at frequencies such that the wall is always this close or closer. In fact, the pressure pulse doesn't reflect just once, but many times, depending on the diameter of the pipe. Thus the narrower the pipe, the higher the impedance.

The cellular picture confirms that short-wavelength sound will escape the pipe more readily than does long-wavelength sound. The frequency is higher for the shorter wavelength, so a cell just inside the pipe may not get an in-phase, reinforcing reflection from the walls in time to increase its impedance. It acts more like a free cell and thus doesn't notice much change as it encounters cells outside the pipe: little impedance mismatch, and little reflection. This is exactly what is seen in the *Ripple* simulation in figure 1.8, where a wave train of the same wavelength is traveling down a narrow and a wide pipe (right). After the encounter with the open end, much stronger reflection is seen inside the narrow pipe, and stronger transmission is seen outside the wide pipe (even accounting for the fact that there was more wave energy in the big pipe to begin with). Take note of the wavelength of the wave compared to the pipe diameter in both cases.

If a pipe suddenly becomes narrower, or wider, there is a corresponding abrupt impedance change (mismatch) at the junction of the two sections of pipe. If a positive pressure pulse is traveling from a wider to a narrower pipe, a positive pressure pulse returns from the junction, reflecting part of the energy. If instead it encounters a wider pipe, a negative pressure pulse reflects part of the energy (figure 1.9).