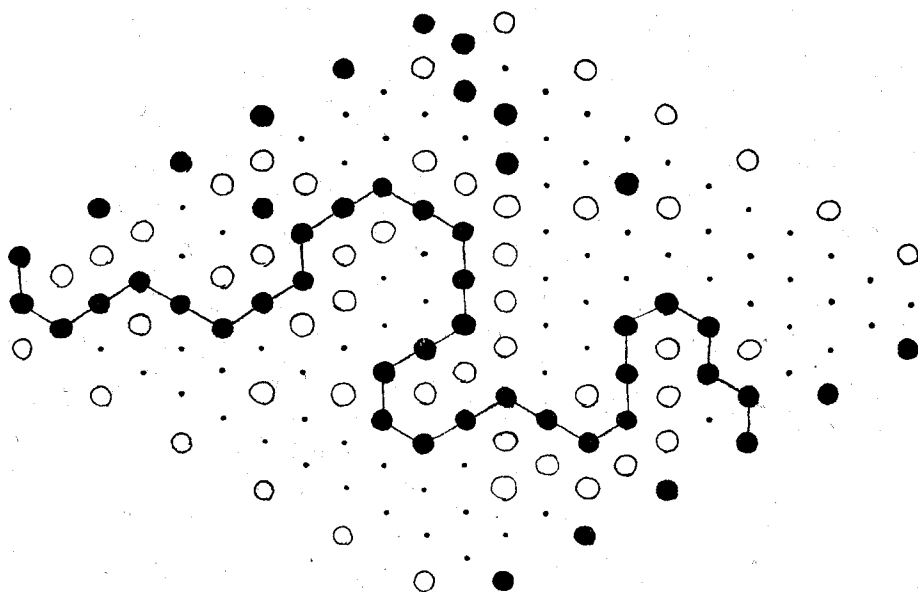


CON-TAC-TIX™

A
PIET HEIN
Game



Produced in Denmark
by
SKJØDE
of
SKJERN
exclusively for
PARKER BROTHERS, INC.,
SALEM, MASS., USA

Condensed from **SCIENTIFIC AMERICAN**
 and from »Mathematical Puzzles and Diversions«
 by Martin Gardner

It is something of an occasion these days when someone invents a mathematical game that is both new and interesting. Such a game is CON-TAC-TIX, introduced 15 years ago at Niels Bohr's Institute for Theoretical Physics in Copenhagen. It may well become one of the most widely played and thoughtfully analyzed new mathematical games of the century.

CON-TAC-TIX is played on a diamond-shaped board. Two opposite sides of the diamond are marked black; the other two sides are white. The holes at the corners of the diamond belong to either side. One player has a supply of black balls; the other, a supply of white balls. The players alternately place one of their balls in any one of the holes, provided the hole is not already occupied by another ball. The objective of Black is to complete an unbroken chain of black balls between the two black sides. White tries to complete a similar chain of white balls between the white sides.

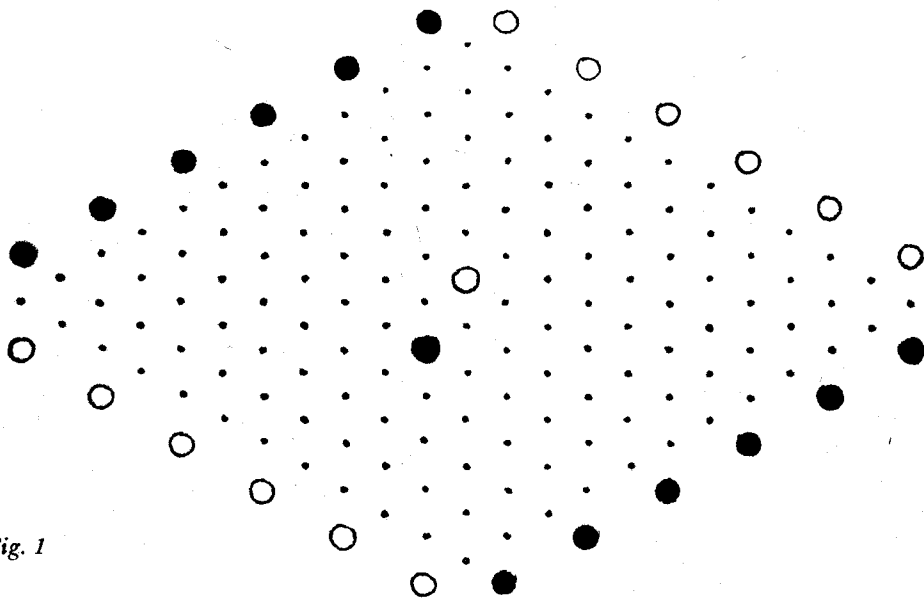


Fig. 1

The board. The two rows of black circles, and two rows of white circles indicate the sides of the board which Black and White respectively must try to connect. On the board itself these four rows are indicated by dark and light spots. The two circles in the center of the board show one of the possible openings of the game, White's move and Black's counter-move.

The chain may freely twist and turn; an example of a winning chain is shown on the front page. The players continue placing their balls until one of them has made a complete chain. The game cannot end in a draw, because one player can block the other only by completing his own chain. These rules are simple, yet CON-TAC-TIX*) is a game of surprising mathematical subtlety.

On the front page Black has formed a winding but unbroken chain between the two sides of the board – thereby winning the game.

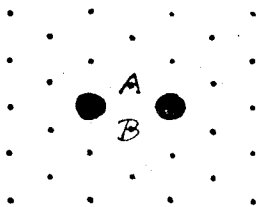


Fig. 2

It's worth noticing from the start that if you occupy two places with two unoccupied places in between – as Black does here – the connection between the two occupied places is ensured since, if your opponent occupies place A, you then occupy place B, or vice versa.

CON-TAC-TIX was invented by Piet Hein, who must surely be one of the most remarkable men in Denmark. Piet Hein began his career at the Institute for Theoretical Physics; then his industrial inventions switched him to engineering, where he remained until the Germans invaded Denmark in 1940. After the war he became well known as a writer on scientific and other topics. He is also known as the author of numerous volumes of epigrammatical poems called grooks.

*

The game of CON-TAC-TIX occurred to Piet Hein while he was contemplating the famous four-color theorem of topology. The theorem, as yet unproved, is that four colors are sufficient to make any map so that no two countries of the same color have a common boundary. Piet Hein introduced the game in 1942 at the Niels Bohr Institute. The same year a leading Danish newspaper published an account of the game; it soon became enormously popular in Denmark (under the name of Polygon). Pads on which the game could be played with a pencil were sold, and for many months the newspaper ran a series of CON-TAC-TIX problems, with prizes for the best solutions. Albert Einstein who was passionately interested in games had since 1948 Piet Hein's CON-TAC-TIX on a shelf in his study.

*) Earlier called Polygon or Hex.

One of the best ways to learn the subtleties of CON-TAC-TIX is to play the game on a field with a small number of holes. When the game is played on a two-by-two board (four holes), the player who makes the first move obviously wins. On a three-by-three board the first player wins easily by making his first move in the center of the board. Because Black has a double play on both sides of his ball, there is no way in which his opponent can keep him from winning on his third move.

On a four-by-four board things begin to get complicated. The first player is sure to win if he immediately occupies any one of the four holes in the central vertical line (fig. 3 B). If he makes his opening play elsewhere, he can always be defeated. An opening play in hole 2 or 3 from the top in the central vertical line insures a win on the fifth move; an opening play in hole 1 or 4, a win on the sixth move.

On a five-by-five board it can still be shown that if the first player immediately occupies the hole in the center, he can win on his seventh move. On larger fields the analysis becomes enormously difficult. Of course the standard twelve-by-twelve board introduces such an astronomical number of complications that a complete analysis seems beyond the range of human computation.

Game theorists find CON-TAC-TIX particularly interesting for the following reason. Although no »decision procedure« is known which will assure a win on a standard board, there is an elegant *reductio ad absurdum* »existence proof« that there is a winning strategy for the first player on a field of any size! (An existence proof merely proves the existence of something without telling you how to go about finding it.)

A few words about general strategy in playing CON-TAC-TIX. Quite a number of readers wrote that they were disappointed to discover that the first player has an easy win simply by taking the center hole, then extending a chain of adjacent holes toward his two sides of the board. They argued that since he always has a choice of two holes for the next link in the chain, it would be impossible to block him. Of course they failed to play long enough to discover that chains can be blocked by taking holes that are not adjacent to the ends of the chain. The game is much subtler than it first appears. Effective blocking often involves plays that seem to have no relationship to the chain that is being blocked.

A more sophisticated strategy is based on the following procedure. Play first in the center, then seek to form on each of your sides a chain of separated links that are either horizontal or perpendicular to your own two sides. If your opponent checks you vertically, you switch to a diagonal play and if he checks you diagonally, you switch to vertical. Of course, once you succeed in joining your two sides with a disconnected chain on which each missing link is a double play, you cannot be blocked. This is a good strategy to play on novices, but it can be countered by proper defensive moves.

Still another strategy provided the basis of a CON-TAC-TIX machine constructed by Claude Shannon and E. F. Moore, both at that time on the staff of Bell Telephone Laboratories. Here is Shannon's description of the device from his article on »Computers and Automata« in the *Proceedings of the Institute of Radio Engineers*, Vol. 41, October 1953:

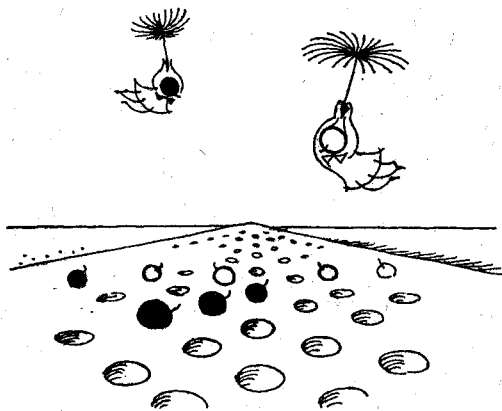
After a study of this game, it was conjectured that a reasonably good move could be made by the following process. A two-dimensional potential field is set up corresponding to the playing board, with white pieces as positive charges and black pieces as negative charges. The top and bottom of the board are negative and the two sides positive. The move to be made corresponds to a certain specified saddle point in this field.

To test this strategy, an analog device was constructed, consisting of a resistance network and gadgetry to locate the saddle points. The general principle, with some improvements suggested by experience, proved to be reasonably sound. With first move, the machine won about 70 per cent of its games against human opponents. It frequently surprised its designers by choosing odd-looking moves which, on analysis, proved sound. We normally think of computers as expert at long, involved calculations and poor in generalized value judgments. Paradoxically, the positional judgment of this machine was good; its chief weakness was in end-game combinatorial play. It is also curious that the CON-TAC-TIX player reversed the usual computing procedure in that it solved a basically digital problem by an analog machine.

There are a number of variations on the basic theme of CON-TAC-TIX, including a »negative« version in which each player tries to force his opponent to make a chain. According to a clever proof devised by Robert Winder, at the University of Princeton, the first player can always win this game on a board which has an even number of holes on a side, and the second player can always win on a board with an odd number.

•

After the reader has played CON-TAC-TIX for a while, he may wish to tackle three problems devised by Piet Hein. These are set forth in the three illustrations of Figure 4. The objective in all three problems is to find the first move that will insure a win for White.



Problems worthy
of attack
prove their worth
by hitting back.

PIET HEIN

Problems

Three Con-Tac-Tix problems: It is White's turn to move. There is, in each problem, only one correct first move which will ensure White's victory. Which?

Solutions are shown in figure 4. But don't look till you have tried.

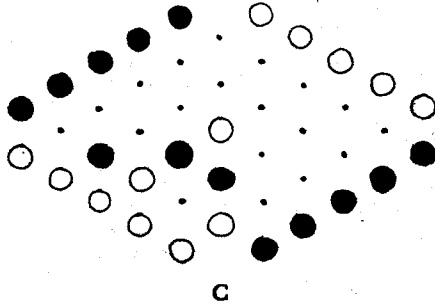
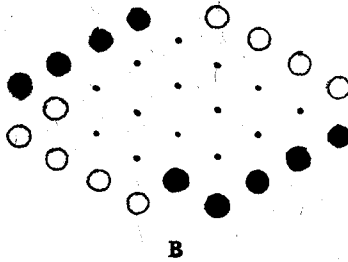
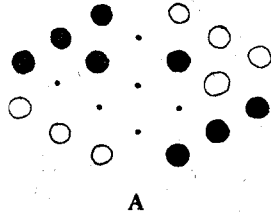
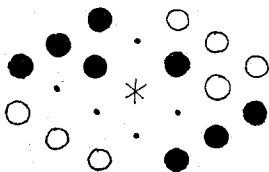


Fig. 3

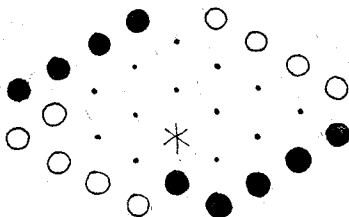
Solutions

Don't look!

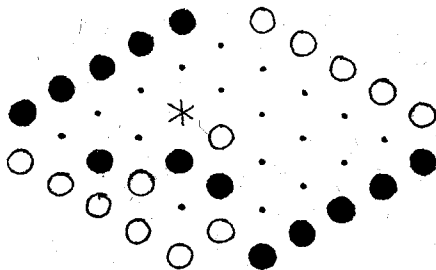
The one correct first move for White is indicated by the crosses.



A



B



C

Fig. 4